Three Essays on the Efficiency of Selected Financial Markets

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Introduction

The aim of this thesis is to analyze the efficiency of selected financial markets. The first and the second paper concern the currency market. The first paper presents an inefficiency in the currency market, as the uncovered interest rate parity seems to fail. This failure leads to the profitability of the carry trade. In the second paper, the "opposite" result is presented, as the uncovered interest rate parity does in fact hold. The important difference is that it holds for long-term interest rates, but not for short-term ones. The solution to this puzzle is that long-term interest rates are a bad proxy for the future short-term interest rate, and the failure of this expectation theory arises from the profitability of the carry trade working with short-term interest rates. The third paper analyzes the cash equity and futures market. The analysis shows that these markets are quite efficient in every kind of view, except for certain small inefficiencies during the financial crisis. However, this efficiency leads to a tax arbitrage opportunity, as the markets have different tax proprieties.

The first paper makes the basic assumptions that exchange rates follow a random walk, which proves to be a reasonable assumption in the empirical section. Given this, the uncovered interest rate parity is said to be failing. With this assumption, the carry trade – borrowing in low-yielding currencies and lending in high-yielding currencies – is expected to be profitable. However, the traditional carry trade contains a high crash risk. To control this risk, I used a mean-variance optimization approach with interest differentials as expected return, and historical volatility as forecast for the expected volatility. The crash risk can be diversified away with a mean-variance optimized portfolio, as mean-variance outperforms an equal weighted 1/n portfolio on a risk-adjusted basis. The diversified carry trade turns out to have been a surprisingly low-risk strategy over the last 20 years, with Sharpe ratios between 0.8 and 1.6 depending on the design of the strategy. The results are robust over the investment horizon, the currency universe, and the optimization methodology. UIP can be rejected for all strategies.

The second paper analyses the well-documented failure of the uncovered interest rate parity over short horizons. While this failure can be confirmed with short-term interest rates, uncovered interest rate parity holds for long-term interest rate returns, even over short horizons. The distinctive point is that the return of the 1-month interest rate over one month is exactly that interest rate (respectively 1/12 of it), while the return of an investment in a long-term interest rate, which is an investment in a bond or an interest rate swap, is dominated by the change in the interest rate during that month. The puzzle is still why the uncovered interest rate parity fails for maturities between one month and six months, but holds for maturities between one year and 30 years. The solution to this puzzle is that the expectation theory of the interest rate term structure also fails, as long-term interest rates are a bad predictor for future short-term interest rates. Because of the expectation theory failure, UIP cannot hold for all interest rate maturities, and empirical analysis definitively shows that long-term maturities are better able to explain short-term exchange rate movements.

The third paper compares the equity futures with the cash market. The futures market proves to be highly efficient, with hardy arbitrage opportunities. Transaction costs in futures are on average one third of those in the cash market, but ongoing costs are higher due to rollover costs. Even the future-implied market estimation of futures dividends is quite efficient, with an exception in 2008, when either the market had underestimated the subsequent dividends, or its increased risk aversion caused market values of futures dividends that were too low. Despite this efficiency, investments in the futures or cash market are not identical, as the cash market has a tax disadvantage. In the cash market, dividends are taxed with the withholding tax, and many investors cannot reclaim them in foreign countries. In the futures market, dividends are not taxed, as they are earned indirectly within the futures price. Therefore, for tax-exempt investors, long-term investments should be conducted in the cash market, while investors who cannot reclaim foreign withholding taxes should only invest in futures. Short-term investments in futures are superior for all investors, due to the lower transaction costs.

Part I

Currency Portfolio Optimization under the Random Walk Hypothesis

Currency Portfolio Optimization under the Random Walk Hypothesis

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Abstract

The aim of this paper is to address risk in carry-based currency portfolios. I take the interest differentials of currencies as the return forecast, assuming that currency movements follow a random walk. The historical volatility serves as basis to forecast the variance. Traditional carry trades tend to be profitable, but contain a high crash risk. The approach presented herein is a mean-variance optimization of a currency portfolio in order to obtain the best return-to-risk ratio. The crash risk can be diversified away with a mean-variance optimized portfolio, as mean-variance outperforms an equal weighted 1/n portfolio on a risk-adjusted basis. The diversified carry trade turns out to have been a surprisingly low-risk strategy over the last 20 years, with Sharpe ratios between 0.8 and 1.6 depending on the design of the strategy and the time period. This means the uncovered interest rate parity can be rejected. The results are robust over the investment horizon, the currency universe, and the optimization methodology.

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I.1 Introduction

The foreign exchange market is by far the biggest financial market in the world, with a daily turnover rate of $4 trillion,[1] and is characterized by high liquidity and low transaction costs. Despite those features, there is still no consensus on which model can predict exchange rate movements over the short time horizon (<1 year).

There are two major advocators of exchange rate models: on one hand, there are fundamental models that try to predict the future exchange rate with fundamental data, while on the other hand the random walk model says that exchange rate movements are not predictable. Three fundamental models are well established in the recent literature.

The uncovered interest rate parity model (UIP)[2], first proposed by Fisher [1896], states that a currency with a high interest rate should depreciate compared to a low interest rate currency ceteris paribus, in order to exactly offset the gains received by the higher yield. The idea behind UIP is that an unattractive currency that is expected to depreciate has to offer a higher interest rate in order to offset the depreciation. Gassel [1918] proposed the concept of purchasing power parity (PPP) to predict exchange rates. PPP is based on the law of one price. When the price level in the domestic country is lower than it is in the foreign country, the exchange rate of the domestic country should appreciate compared to the foreign country. A more recent macroeconomic model that includes exchange rate theory is the Mundell-Fleming model, which was developed independently by Mundell [1963] and Fleming [1962]. The Mundell-Fleming model is an extension of the IS-LM model for an open economy. In this model, the exchange rate depends on the current account and the capital account.[3] A current account surplus produces an upward pressure on the domestic currency. The domestic currency appreciates until the surplus vanishes. Thus, a current account surplus predicts an appreciation of the domestic currency.

The random walk model was first mentioned by Meese and Rogoff [1983] in association with exchange rate models. In the random walk model, the expected change of the exchange rate is zero. Hence, the current exchange rate is the best estimation for the future exchange rate, as supported by Meese and Rogoff with empirical data for short time horizons. Recently, Engel and West [2005] and Cheung, Chinn, and Pascual [2005] confirmed these findings with data that are more recent, as did many others in the meantime.

[1] Compare to "Foreign Exchange and Derivatives Market Activity in 2010", BIS
[2] UIP is also known as international Fisher effect
[3] The capital account is the opposite of the current account; a current account deficit is always financed by a capital account surplus.

The basic assumption in this paper is that exchange rates follow a random walk, since empirical evidence strongly supports this theory.

The empirical failure of fundamental models induces a puzzle in the covered interest rate parity (CIP). In CIP, the forward exchange rate is equal to the spot exchange rate minus the interest rate difference between the currencies of the forward contract. In contrast to UIP, CIP must hold in the absence of arbitrage. If the actual forward rate for a higher-yielding currency were equal to the spot rate, an arbitrageur could buy the currency at the spot rate and sell it simultaneously at the forward rate; hence, there is no exchange rate risk. However, the arbitrageur will earn the interest yield of the high-yielding currency, instead of that of the low-yielding currency. In fact, Taylor [1987] shows that CIP holds in general. Thus, an investor can buy a high interest rate currency against a low interest rate currency for a forward rate that is lower than is the expected future spot rate. This is called a carry-trade. The carry trade generates an expected profit in the amount of the interest differential, given the random walk assumption of the exchange rate.

Fama [1984] was the first to observe this forward premium puzzle. Subsequently, many studies were published, showing profitable trading strategies that exploited this puzzle. Sarno and Taylor [2002] surveyed the research in this area. Other studies tried to explain part of the puzzle via the peso problem, where rare but extreme events could reverse the returns (see Krasker [1980]). Wu [2007] proposed a nonlinear relationship between spot and forward rates as an explanation. Lustig and Verdelhan [2007] explained higher interest rates with time-varying risk premium factors. Nonetheless, the carry trade strategy has been empirically proven to be profitable.

I.2 Diversification of Currency Risk

The implementation of the carry trade strategy in praxis bears an exchange rate risk, which can be highly risky if only the currencies with the highest and lowest interest rates are considered (see Table 3 in section I.4.5). By forming a portfolio of currencies, the diversification leads to a reduction of the risk, but lowers the expected return. The expected return is maximal if the highest interest rate currency is bought and the lowest is sold. Thus, there is a tradeoff between risk and return, which can be analyzed via a mean-variance analysis.

At this stage, various approaches to forming a diversified currency portfolio exist. Levy [1981] used an ex-post covariance to build a mean-variance portfolio, allowing for negative weights. Mettler, Thoeny, and Schmidt [2010] formulated an interest-weighted portfolio, where each currency is weighted with the difference of its interest rate minus the mean interest rate of all currencies, allowing for positive and negative weights. Baz, Breedon, Naik, and Peress [2001] and Huang [2002] followed a mean-variance approach by minimizing the variance given a target return. For the optimization process, they used a periodically estimated covariance matrix, and interest rate differentials as estimated returns.

The focus in this paper is a practical approach to optimizing a currency portfolio. I minimize the risk for a given return. The implementation and backtest is done with currency forwards, which means very little collateral is necessary[4]. The risk minimization for a given return gives the best return-to-risk relation. In other words, the resulting portfolio will be on the steepest point of the estimated efficient frontier. Thus, by definition, leverage of this strategy will always be equal or superior to all other possible strategies on a risk-return perspective, as long as I forecast the steepest point on the efficient frontier accurately. Hence, I use an ex ante estimated covariance to optimize the portfolio. I show different models to estimate the covariance and their implications for the estimation results. The year 2008 is also included in the dataset, as one of the biggest world economic crises during the last decades, which offers new, additional insights. The performance of the resulting currency portfolio is much less volatile than existing approaches, and shows robustness in turbulent years. The risk-adjusted performance is significantly higher than those in existing approaches, and UIP can be rejected for all strategies.

The idea is a sequel to the study of Baz, Breedon, Naik, and Peress [2001], who had already produced convincing results for a currency basket of five currencies. The key differences from their study are:

- A longer time horizon: 1987-2010 instead of 1989-1999
- A financial crisis in the backtest period
- A broader currency basket: 11 currencies instead of 5
- Forward implied interest rates instead of LIBOR

I.2.1 Single Carry Trade

The exchange rate is the price of one currency relative to another currency. I define the spot exchange rate, S_t, as the price of one unit of foreign currency in the domestic currency. Thus, an increase in S_t means the foreign currency appreciates.

How does a single carry trade between two currencies work? Let r_t be the one-period foreign interest rate for borrowing and lending, and r_t^d the corresponding risk-free rate in the domestic currency. Assuming that the foreign interest rate is higher than is the domestic one, a carry trade is supposed to be prosperous if the investor borrows in the domestic currency and invests in the foreign currency. For one unit of domestic currency, the investor gets $1/S_t$ units of the

[4] At the Zuercher Kantonalbank (ZKB), depending on the risk of a currency, around 10% of a currency forwards nominal has to be deposited as collateral.

foreign currency. After one period, the investor receives $(1 + r_t)/S_t$ for the investment in the foreign currency and repays the loan plus interest costs $1 + r_t^d$ in the domestic currency. In the domestic currency, this is

$$\frac{S_{t+1}}{S_t}(1+r_t) - (1+r_t^d).$$

While r_t, r_t^d and S_t are known at t, the future value of S_{t+1} is unknown. However, under the random walk assumption, the expected exchange rate in t+1 is equal to the one in t

$$E(S_{t+1}) = S_t$$

and the expected profit from the investment is

$$r_t - r_t^d, \tag{1}$$

as long as $r_t > r_t^d$, the expected profit of this carry trade, is positive.

The risk of the carry trade is measured by the variance

$$Var\left(\frac{S_{t+1}}{S_t}(1+r_t) - (1+r_t^d)\right) = Var\left(\frac{S_{t+1}}{S_t}\right)(1+r_t)^2$$

The Sharpe ratio is the excess return above the risk-free rate, divided by the volatility of the excess return. Thus, the Sharpe ratio of the carry trade is

$$S = \frac{r_t - r_t^d}{\sqrt{Var\left(\frac{S_{t+1}}{S_t}\right)}(1+r_t)} \tag{2}$$

I.2.2 Portfolio of Optimal Carry Trades

For a portfolio of currencies, it is reasonable to define all exchange rates in terms of the investor's domestic currency[5]. Let us assume that the investor has a portfolio of n foreign currency instead of just 1. The interest rates for those currencies are r_t^i for i=1..n and r_t^d. The interest rates of those currencies are r_t^i for i=1..n, and r_t^d is the interest rate of the domestic currency. For n foreign currencies and the domestic currencies, n exchange rates exist between the foreign and the domestic currency. These exchange rates can be considered to be investments with the usual risk and return characteristics. For $r_t^i > r_t^d$, the expected return of the investment is positive, while it is negative for $r_t^i < r_t^d$. The risk can be measured by the volatility of the exchange rate, and by the correlation between the exchange rates.

[5] The optimal long-short portfolio is almost "domestic" currency independent; nevertheless, it is better to focus on one domestic currency for the consistent illustration of the other currencies. However, the risk-free interest rate depends directly on the domestic currency.

The weight of each currency in the portfolio is $\mathrm{w_t^d}$ for the domestic currency and $\mathrm{w_t^i}$ for the foreign currencies. Under the constraint that the sum of all weights is 1, $\mathrm{w_t^d}$ can be replaced by $1-\sum_{\mathrm{i=1}}^{\mathrm{n}}\mathrm{w_t^i}$. The weights $\mathrm{w_t^i}$ are unconstrained, and can also be negative. A negative weight means the investor is borrowing in this currency, whereas a positive weight means that he or she is lending in that currency.

The mean-variance approach is still the most widespread methodology in practice. Despite some criticism, it is a reliable approach to obtaining a well-balanced portfolio (Kritzman [2011]).

The expected portfolio return under the random walk hypothesis is

$$\mu_t^p = E\left[\left(1-\sum_{i=1}^{n} w_t^i\right) r_t^d + \sum_{i=1}^{n} w_t^i \frac{S_{t+1}^i}{S_t^i} r_t^i\right] = \left(1-\sum_{i=1}^{n} w_t^i\right) r_t^d + \sum_{i=1}^{n} w_t^i r_t^i = r_t^d + \sum_{i=1}^{n} w_t^i (r_t^i - r_t^d)$$

In the vector notation (see Fabozzi, Kolm, Pachamanova, and Focardi [2007]), the vector of returns is $\mathrm{r_t} = (\mathrm{r_t^1}, \mathrm{r_t^2}, .., \mathrm{r_t^n})$, vector of weights is $\mathrm{w_t} = (\mathrm{w_t^1}, \mathrm{w_t^2}, .., \mathrm{w_t^n})$ and the vector of ones is $\iota = (1,1, .., 1)'$. Under this vector notation, the return is

$$\mu_t^p = r_t^d + w_t'(r_t - r_t^d \iota). \tag{3}$$

The covariance of returns for the investment in the foreign currencies i and j is

$$Cov\left[\frac{S_{t+1}^i}{S_t^i}(1+r_t^i), \frac{S_{t+1}^j}{S_t^j}\left(1+r_t^j\right)\right] = Cov\left[\frac{S_{t+1}^i}{S_t^i}, \frac{S_{t+1}^j}{S_t^j}\right](1+r_t^i)(1+r_t^j) \tag{4}$$

since the interest rate $\mathrm{r_t^i}$ is assumed to be risk-free. Denoting the variance-covariance matrix at time t by Σ_t, the portfolio variance is

$$(\sigma_t^p)^2 = w_t' \Sigma_t w_t$$

If the investor demands a return of μ_0, then the portfolio optimization problem at time t is as follows:

$$\begin{aligned} &\min_{w_t} && w_t' \Sigma_t w_t \\ &s.t. && r_t^d + w_t'(r_t - r_t^d \iota) = \mu_0. \end{aligned}$$

The optimal portfolio weights solving this optimization problem are

$$w_t^* = \frac{\mu_0 - \quad r_t^d}{(r_t - r_t^d \iota)' \Sigma_t^{-1} (r_t - r_t^d \iota)} \Sigma_t^{-1} (r_t - r_t^d \iota). \tag{5}$$

These weights are identical to the solution to the Sharpe ratio-maximization problem,

$$\begin{aligned} &\max_{w_t} && \frac{w_t'(r_t - r_t^d \iota)}{\sqrt{w_t' \Sigma_t w_t}} \\ &s.t. && r_t^d + w_t'(r_t - r_t^d \iota) = \mu_{0,} \end{aligned}$$

because the optimal unconstrained weights, w_t^*, will always maximize the Sharpe ratio. All components, except Σ_t, are known at t. Σ_t is the only uncertain variable and has to be estimated.

The Sharpe ratio can also be maximized with this maximization problem:

$$\begin{array}{ll} \underset{w_t}{max} & \dfrac{w_t'(r_t - r_t^d \iota)}{\sqrt{w_t' \Sigma_t w_t}} \\ \mathrm{s.\,t.} & \sqrt{\mathrm{w}_t' \Sigma_t \mathrm{w}_t} = \sigma. \end{array}$$

In practice, investors often want to hedge the currency risk, but are reluctant because some currencies are costly to hedge due to the higher interest rates. I suggest that those investors should use the Sharpe ratio-maximization formula with additional constraints for the currencies to hedge[6]. σ has to be set at the desired level of volatility of the currency portfolio.

I.2.3 Estimation of the Variance-Covariance Matrix

There are numerous models for the estimation of the variance-covariance matrix. I used the most common models which, in my opinion, are the GARCH (generalized autoregressive conditional heteroskedasticity model), the EWMA (exponentially weighted moving average model), and the traditional moving average model. All models use historically realized returns to estimate the future variance-covariance matrix. Andersen [2006] compared the different models and surveyed the related literature.

Let d be the number of days from the past that are used for the covariance estimation. The estimated covariance between the returns of the foreign currencies i and j in the moving average model is:

$$E\left(Cov_t^{i,j}\right) = \frac{\sum_{x=t-d}^{t} Cov\left[\frac{S_x^i}{S_{x-1}^i}, \frac{S_x^j}{S_{x-1}^j}\right]}{n} \left(1 + r_t^i\right)\left(1 + r_t^j\right). \tag{6}$$

The expected covariance at date t is the equally weighted average covariance of the historically realized returns from the past d days.

In the EWMA model, the more recent return has an exponentially increasing weight. This is defined as

[6] While the standard approach of risk minimization for a given return of risk delivers only the maximal Sharpe ratio in the event of there being no other constraints, the Sharpe ratio maximization delivers the highest Sharpe ratio for every optimization problem.

$$E\left(Cov_t^{i,j}\right) = \left[\lambda\ Cov\left[\frac{S_t^i}{S_{t-1}^i}, \frac{S_t^j}{S_{t-1}^j}\right] + (1-\lambda)\ E\left(Cov_{t-1}^{i,j}\right)\right](1+r_t^i)(1+r_t^j) \qquad (7)$$

where λ is a constant between 0 and 1.

The generalized autoregressive conditional heteroskedasticity model (GARCH (1, 1)) is defined as

$$E\left(Cov_t^{i,j}\right) = \left[\gamma V_L + \alpha\left(\frac{S_t^i}{S_{t-1}^i} - 1\right) * \left(\frac{S_t^j}{S_{t-1}^j} - 1\right) + \beta\ E\left(Cov_{t-1}^{i,j}\right)\right](1+r_t^i)(1+r_t^j) \qquad (8)$$

where α, β and γ are constants, and V_L is the long-term variance. In addition, $\alpha + \beta < 1$ is required for a stable process.

The multivariate GARCH model can be expressed by the dynamic conditional correlation (DCC) model of Engel (2002).

$$Cov = D_t R_t\ D_t$$

D_t is a diagonal $n \times n$ matrix, where every component is defined by a univariate GARCH (1, 1) process as defined in (9). R_t is the correlation matrix and is defined as follows:

$$R_t = diag\{Q_t\}^{-1} Q_t diag\{Q_t\}^{-1}$$

and

$$Q_t = \left(1 - \sum\nolimits_1^m \alpha_m - \sum\nolimits_1^n \beta_n\right)\bar{Q} + \sum\nolimits_1^m \alpha_m\left(\varepsilon_{t-m} - \varepsilon'_{t-m}\right) + \sum\nolimits_1^n \beta_n Q(t\text{-}n)$$

$\bar{Q}$ is the unconditional covariance matrix of ε_t and of ε_t is D_t^{-1} multiplied by μ_t^p.

I.2.4 Benchmark Carry Trade Strategies

For the comparison of the optimization strategy, I used three benchmark strategies to determine the weights, w_t^i, of all currencies, including the domestic currency. For simplicity, the sum of the absolute value of the weights is normalized to one and the weight of the domestic currency is the resulting weight +1. These strategies are commonly used in the existing literature.

The simple carry strategy (SI) has +0.5 weight for the currency with the highest interest rate, and -0.5 for the lowest yielding currency. This strategy maximizes the expected return, but is probably also the riskiest of the strategies, since there is no diversification.

$$w_t^i = \begin{cases} +0.5 & if\ r_t^i = max(R_t) \\ -0.5 & if\ r_t^i = min(R_t) \\ 0 & if\ min\ (R_t) < r_t^i < max(R_t) \end{cases} \qquad (9)$$

The equally weighted carry strategy (EW) has a weight of $+\frac{1}{n}$ for each of the $\frac{n+1}{2}$ currencies among the $\frac{n+1}{2}$ highest interest rates, and a weight of $-\frac{1}{n}$ for each of the other $\frac{n+1}{2}$ currencies among the lowest interest rates. This strategy is also known as the 1/n approach. The rank of the highest interest rate is 1 and the lowest is n. This strategy is risk minimizing under the condition that the exchange rates are uncorrelated and are equally risky.

$$w_t^i = \begin{cases} +\dfrac{1}{n} & if\ rank\ (r_t^i)|(R_t) \leq \dfrac{n+1}{2} \\ -\dfrac{1}{n} & if\ rank\ (r_t^i)|(R_t) > \dfrac{n+1}{2} \end{cases} \tag{10}$$

The interest weight carry strategy (IW) has a weight for each currency that is proportional to the interest rate difference between its interest rate and the average interest rate of all currencies. This strategy is a compromise between return maximizing and risk minimizing, given that all exchange rates are uncorrelated and are equally risky.

$$w_i(t) = \frac{(r_t^i - \bar{r}_t)}{\sum_{i=1}^{n} abs(r_t^i - \bar{r}_t)} \tag{11}$$

The term $\sum_{i=1}^{n} \mathrm{abs}(\mathrm{r}_t^i - \bar{\mathrm{r}}_t)$ normalizes the sum of the absolute values of all weights to one.

I.3 Carry Trade for Practitioners

I.3.1 Carry Trade Implementation

In practice, the implementation of the carry trade is typically done through currency forwards. A currency forward is a contract to buy a specific amount of one currency, and to sell another currency at a specific date in the future at a price that is fixed today. There is no initial cash flow. The forward price, also called the outright, consists of the current spot rate and the forward points for the time until delivery.

The forward exchange rate is:

$$F_t = S_t \frac{(1 + r_t^d)}{(1 + r_t^i)}$$

where F_t is the current forward rate, r_t^d is the interest rate corresponding to the domestic currency for the time to expiry of the currency forward, and r_t^i is the interest rate of the foreign currency. The forward points, the difference between the spot rate and the forward rate, are defined as:

$$Forward\ Points = \ S_t \left[\frac{(1 + r_t^d)}{(1 + r_t^i)} - 1\right]$$

The market for the forward points is a separate market for trading interest differentials, and is organized as follows:

For the customer (i.e. the investor), the market is not separated, since s/he will get one price for the currency forward. However, for the counterparty of the customer (i.e. the bank), there are two separate markets in the interbank market. For one trade with the customer, the bank will buy the currency on the spot market, through selling the other currency. This is done by the spot trading desk. Let us assume that the customer buys AUD and sells USD one year forward. The bank will then buy AUD and sell USD on the spot market today. Furthermore, the bank has to refinance the short USD position daily, and earns interest for the AUD. However, there is a duration mismatch between the demand obligation to the customer and the position of the bank. Therefore, the bank (as performed by the forward trading desk) will make an FX Swap[7], which buys USD and sells AUD today, but has to obligation to buy AUD back in a year through selling the USD. The difference between today's price and the one in the following year are the forward points (in the words of the customer), or the swap spread (in the words of the bank).

The forward points depend on the interest rate difference between the two currencies, the time to maturity, and the spot rate price. Contract specifications are custom, but standard is the nominal exchange at the delivery date with no margining meanwhile. The profit and loss is realized at the delivery date.

Due to the positive interest rate difference in the carry trade, the forward price is lower than is the current spot price. If the spot rate does not change, the profit is the interest rate difference, which corresponds to the forward points. The interest rate difference can be earned subsequently by rolling-over the contract as long as the interest rate difference remains positive.

Numerical example for the client: The client sells 1 million USD and buys AUD one year forward. The current spot rate is 0.95 AUD/USD. The 1-year interest rate is 5% for AUD and 1% for USD. The forward points are -0.0362 (= (1.01/1.05-1)*0.95), and the forward rate is 0.9138; thus, 1'094'320 AUD are bought forward. Assume the spot rate is still 0.95 AUD/USD in one year. Then, in one year, the contract delivers 1'094'320 AUD by paying 1 million USD. The AUD can be sold at 0.95, which is 1'039'604 USD. The profit is 39'604 USD.

Numerical example for the bank: The banks sells 1'039'604 USD, and buys 1'094'320 AUD on the spot market today (spot rate 0.95). On the swap market, the following transaction takes place: FX Swap AUD-USD from today to one year at the swap rate of -0.0362. The FX Swap consists of two transactions, sell 1'094'320 AUD today and buy 1'039'604 USD at 0.95, and buy 1'094'320 AUD and sell 1'000'000 USD in a year at 0.9138.

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[7] The majority of the deals will run through FX swaps (see Biz 2010). However, the forward trader also has the option of buying an interest rate future, or and cash bond.

Alternatively, the bank could sell 1'000'000 USD and buy 1'052'632 AUD on the spot market today (spot rate 0.95). It could then buy an Australian bond that yields 5%. In one year, it would earn 52'632 AUD interest and have 1'105'263 AUD. However, for the 1 million USD, the credit costs are 1%, which is 10'000 USD, respectively expected to be 10'943 AUD (=10'000/0.9138). Therefore, in one year, the bank has 1'094'319 AUD (=1'105'263-10'943), which it has to deliver to the client. The client gives the bank 1'000'000 USD, which it can use to pay back the credit. However, there is a risk that the exchange rate is not 0.9138 in one year and the bank does not pay exactly 10'943 AUD - in the case of the random walk, the 10'000 USD will be only 10'526 AUD. To hedge against this risk, the bank could make a forward deal and sell 10'000 USD today for 10'943 AUD. However, the impact of this risk is small.

I.3.2 Transaction Costs

The following table shows half of the bid-ask spread for one month's currency forwards (source: ZKB trading desk, from their experience with institutional clients[8]):

Table 1: Spot and Forward Transaction Costs

	CHF/USD	EUR/USD	JPY/USD	GBP/USD	AUD/USD
Spot	0.016%	0.013%	0.019%	0.012%	0.025%
Swap	0.004%	0.002%	0.002%	0.003%	0.010%
	CAD/USD	**NOK/USD**	**SEK/USD**	**SGD/USD**	**NZD/USD**
Spot	0.014%	0.025%	0.022%	0.050%	0.060%
Swap	0.005%	0.012%	0.006%	0.002%	0.017%

The table indicates the transaction costs for specific currency spot and currency spot trades. ZKB trading desks are the source for the transaction costs for institutional investors.

If the investors invest in a new currency AUD-USD forward, the transaction costs consist of 0.025% for the spot deal and 0.010% for the swap deal. However, if the AUD-USD forward is at expiry and the bank wants to refresh it (rollover) for one more month, the costs are only 0.010%. This is because, in this case, the currency swap is done by the investors themselves. In the terms of the numerical example above, the bank sells 1'094'319 AUD today and buys it a year later at the same spot rate, plus the forward points. Since the bank sells at the spot rate and buys at the same spot rate (plus the forward points), there is no bid-ask spread on the spot rate, but only on the forward points, namely 0.010%. More precisely, there is no trade on the spot market. If the bank does not want to take up a new position, it will get 1'094'319 AUD and will have to deliver 1'000'000 USD. If the bank does not want to have the AUD in

[8] Smaller clients pay higher spreads, since the transaction fee is included in the spread. Some very large investors probably pay smaller spreads. The figures here are a good proxy for transactions with a volume of between 1 million and 50 million.

the investors' account, it can sell them at the current spot rate and pay half of the spot bid-ask spread, which is 0.025%.

I.3.3 Return Measurement

There are two ways to measure the return on the investment. If no leverage is allowed, the net return (net transaction costs) is measured as the profit in t+1 divided by the invested amount in t which, for a currency portfolio, is:

$$\mu_t^p = \frac{(1 - \sum_{i=1}^n w_t^i) r_t^d + \sum_{i=1}^n w_t^i \frac{S_{t+1}^i}{S_t^i} r_t^i}{\sum_{i=1}^n abs(w_t^i)} - Transaction\ costs^9$$

This ensures that the sum of the weights is not larger than 1. The same in terms of forward prices, is:

$$\mu_t^p = \frac{(1 - \sum_{i=1}^n w_t^i) r_t^d + \sum_{i=1}^n w_t^i \left[\left(1 + \frac{S_{t+1}^i - F_t^i}{S_t^i} \right) (1 + r_t^i) - 1 \right]}{\sum_{i=1}^n abs(w_t^i)} - Transaction\ costs^{10}$$

For the excess return, the risk-free interest rate r_t^d has to be deducted from the net return.

If leverage is allowed, the net return is:

$$\mu_t^p = \left(1 - \sum_{i=1}^n w_t^i \right) r_t^d + \sum_{i=1}^n w_t^i \frac{S_{t+1}^i}{S_t^i} r_t^i - Transaction\ costs \tag{12}$$

Here, the sum of the weights can be larger than 1.

[9] Theoretically, the transaction costs should be included in the optimization formula. However, in order to reduce complexity and since the costs are insignificant, the transaction costs are ignored in the optimization process.

[10] Theoretically, the transaction costs should be included in the optimization formula. However, in order to reduce complexity and since the costs are insignificant, the transaction costs are ignored in the optimization process. The complexity of the transaction cost inclusions arises from the fact that there are initial costs and maintenance costs. The initial must be spread over the future holding period, which is unknown in advance.

I.4 Empirical Currency Portfolio Optimization

I.4.1 Data

I tested the efficacy of the diversified carry trade strategy as follows: I first took the US dollar (USD) as the domestic currency. The basic foreign currency universe consists of the Euro (EUR), the Japanese yen (JPY), the British pound (GBP), the Swiss franc (CHF), the Canadian dollar (CAD), the New Zealand dollar (NZD), the Australian dollar (AUD), the Norwegian krona (NOK), the Swedish krona (SEK), and the Singapore dollar (SGD). This corresponds to the universe of the so-called developed countries, except for the exclusion of the Danish krona. The broad foreign currency universe also considers the Danish krona (DKK), the Hungarian forint (HUF), the South African rand (ZAR), the Mexican peso (MXN), and the Polish zloty (PLN).

Table 2: Descriptive Statistics

	Interest Rates	Exchange Rate Return	Exchange Rate Volatility	Exchange Rate Skewness
JPY	0.13%	2.03%	11.74%	0.78
CHF	1.37%	2.40%	10.97%	0.56
SGD	2.44%	0.92%	5.78%	-0.03
EUR	2.97%	1.17%	10.39%	0.07
CAD	3.61%	2.36%	8.29%	-0.23
SEK	3.62%	1.31%	11.25%	0.08
USD	3.77%	0.00%	0.00%	0.00
NOK	4.47%	1.49%	10.66%	-0.21
GBP	4.79%	0.51%	8.47%	-0.27
AUD	5.57%	2.25%	12.16%	-0.50
NZD	6.52%	1.94%	12.54%	-0.16

This table presents the descriptive statistics for the 11 currencies during the period from December 31 1994 to October 31 2010. The interest rate is the average interest rate in that period. The exchange rate return is the arithmetic average of the monthly change in the exchange rate versus the USD. A positive figure means that this currency has appreciated against the USD. Exchange rate volatility is the annual volatility of the exchange rate versus the USD. Skewness is the skewness of the monthly exchange rate change.

Table 2 reports the summary statistics for the 11 currencies in this data set, sorted by the average interest rate. The average interest rates range from 0.13% for the JPY to 6.52% for the NZD. In addition, the table reports the average, standard deviation, and skewness of the exchange rate changes of each currency versus the USD. On average, all the currencies appreciated against the USD between January 1995 and October 2010, even though four of them had average interest rates exceeding the average interest rate in the U.S. Consequentially, UIP must fail for these currencies. Another readily apparent pattern is that the two highest and the two lowest interest rate currencies are among the five currencies having the highest volatility. A mean-variance analysis strategy will naturally try to minimize its exposure to that extra volatility, if possible.

I used (5) to calculate the weights each month with the data available at this time point, so that it would have been possible to implement it in practice. The required excess return $\mu_0 - r_t^d$ is set to 6.5%, which is the long-term excess return for US equities (for details, see Mehra [2006]). The resulting currency allocation is held for one month. The monthly return is computed with (12). For the monthly estimation of the new covariance in the MA model (6), I used the previous 250 trading days and scaled it by 22 to obtain monthly values. I call this method MA 250. For the GARCH model (8), I used the Dynamic Conditional Correlation Multivariate GARCH model as proposed by Sheppard [2003]. The weighing factors in GARCH are also recalibrated monthly. In EWMA (7), I used a constant weighting factor (λ) of 0.94. RiskMetrics Group [1996] found that λ=0.94 delivers the best results for a broad range of backtests. In EMWA and GARCH, the entire data horizon available at one specific point in time was used with the daily data, starting at the beginning of the dataset for the estimation of the parameters. In addition, I computed the performance with the weights of the benchmark strategies (9)-(11) as a comparison. Here, I scaled the weight so that the expected excess return is 6.5%. For example, if the resulting weights from benchmark (9) generate an expected return of 3.25%, then all weights are multiplied by 2 in order to have an expected excess return of 6.5%.

I.4.2 Data Source

I used two sources for the data, Datastream and Bloomberg. In both cases, I took the German mark (DEM) as a substitute for the EUR for the time period before January 1 1999. All the data are the daily data from every weekday. The timeframe and currency universe were chosen in order to obtain as much data as possible for large countries with liquid currencies. Reliable data existed for only a few currencies before 1985.

In Datastream, I used exchange rates and interest rates. For the exchange rates, I took the middle rates provided by Barclays Bank International from January 1 1985 to October 10 2010. For the interest rates, I used one month's interbank rates from December 31 1986 to October 30 2010[11]. USD, CHF, JPY, GBP and AUD interest rates were provided by Barclays Bank

[11] I needed at least 250 trading days preceding the start of the backtest for the computation of the variance-covariance matrix.

International, NOK was provided by Norgres Bank, EUR by Thomson Reuters, SGD by Development Bank of Singapore, NZD by Datex (New Zealand) until December 31 2003 and by Barclays Bank International afterwards, while SEK was from http://www.riksbank.com until December 31 1992, and from Barclays Bank International afterwards. All rates are offered rates, except for NZD and SGD, which are middle rates. In Bloomberg, I used exchange and forward middle rates from January 1 1994, and forward rates from December 31 1994. Transaction costs for the spot and forward trades of each currency pair are provided by the trading unit of Zuercher Kantonalbank, and are assumed to be constant over time.[12]

I.4.3 Data Manipulation

Missing data in exchange rates were completed with the value of the previous day. For forward rates, a missing value was completed with the value of the previous day multiplied by the spot rate change from the previous day to the current day. This means that, if there were no forward rate at t, then the forward at t would be $F_t = F_{t-1} S_t / S_{t-1}$. This is because forward points are roughly constant, in contrast to the volatile spot rates.

A special case is the EUR/DEM time series. For the computation of the covariance matrix, I used the continued DEM time series, which is exactly the same as the EUR time series multiplied with a fixed conversion factor from the switch data on December 31 1998.

Some days were eliminated from the time series. If the exchange rates of all currencies from one day were identical to those of the previous day, then, for all time series, both Datastream and Bloomberg, the latter day was removed. This was typically the case for important holidays. Overall, 226 weekdays were removed over the 25-year time horizon, which is 9 days per year.

I.4.4 Data Quality

The data quality of the exchange rates is good in both Datastream and in Bloomberg. The only difference is that the Bloomberg data are from 6 pm, while those of Datastream are from 5:30 pm London time.

The data quality of Datastream for forward rates is modest, since there are jumps in the forward points throughout the entire time period. Therefore, I did not consider these rates. However, Datastream has a very long track record of data until 1987 for all interest rates and up to 1985 for the exchange rates. The problem with interbank interest is that it consists of non-binding offered rates that are subject to manipulation (see Snider and Youle [2009]). Furthermore, the covered interest rate parity holds often, but not always. If it does not hold,

[12] The transaction costs are listed in Figure 1.

the carry trade is not implementable analogously with currency forwards. Deviations have been found during the financial crisis of 2007-2009 (see. Coffey, Hrung, and Sarkar [2009]), as well as in the example of the SGD, whose average interbank interest rate over the sample period is 2.44, compared with a 2.94 forward implied interest rate. The effect from the covered interest rate parity violation is large for SGD, since having short SGD generates a negative return. For the other currencies, the effect is small.

The advantage of the Bloomberg data is the high quality forward rates. The data source for the exchange and forward rates is Bloomberg BGN. This data source, according to Bloomberg, is "...totally resistant to manipulation by market participants. The prices in Bloomberg for each currency pair is based on input prices from a subset of Bloomberg's more than 1000 contributors of FX prices."[13] An algorithm eliminates outliers and creates the middle price of each exchange and forward rate. Before 1994, the forward rates in Bloomberg were of low quality. Low quality means that the forward implied interest rate has sudden moves at some data points, which cannot be seen in the related interest rates. The problem is that forward rates have to be measured at exactly the same point in time as the spot rates. If that is not the case, there could be an illusionary trading opportunity. The lack of quality before 1994 is probably because electronic trading emerged with the launch of Reuters 2000 in 1992, and the recording of data was difficult in an OTC market working with phone calls.

I.4.5 Results of Empirical Currency Portfolio Optimization

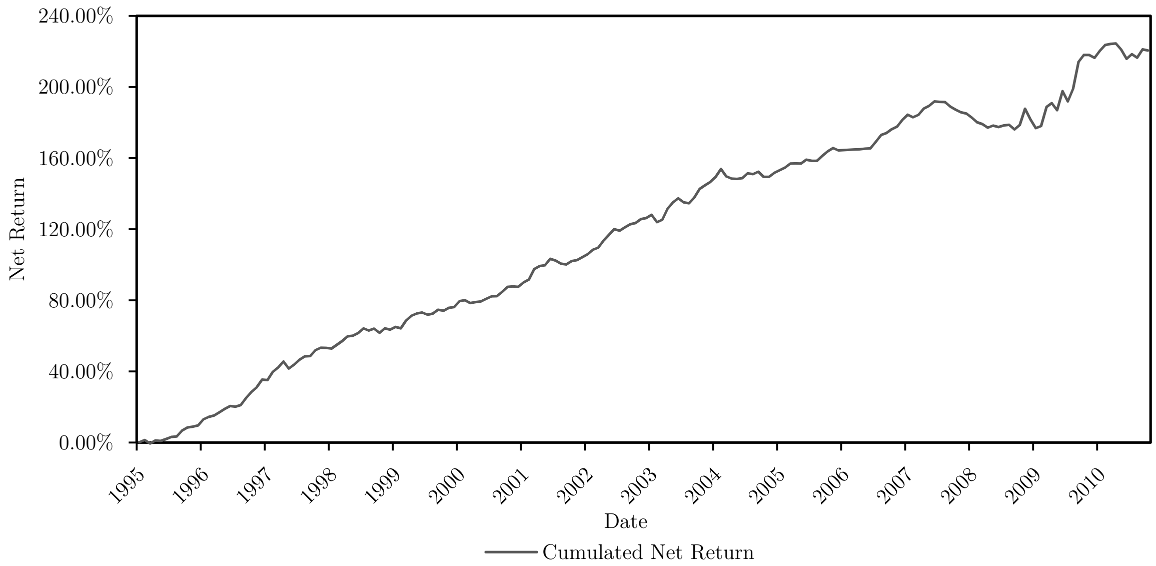
Figure 1 presents the cumulated return, when optimization is applied (5) with monthly rebalancing, using (6) to estimate the covariance matrix with the MA 250 method, and (12) to compute the return net of transaction costs. The data are from Bloomberg, using forward rates to back out the interest rates.

The plot in Figure 1 clearly shows how successful the strategy was. The return was continuously positive, with a major exception from mid-2007 to end of 2008. Moreover, the continuous return indicates that the interest rate difference was the driver of the return.

However, profitability itself does not guarantee superiority to other strategies. One important feature of a successful strategy is beating the risk-free interest rate. Consequently, the risk-free interest rate needs to be deducted from the net return in order to obtain the excess return. Figure 2 plots the excess return of the optimized strategy and the three benchmark strategies (9)-(11), the simple method (SI), the equally weighted method (EW), and interest weighted method (IW).

[13] Source is the description of the Bloomberg BGN rates available on a Bloomberg terminal.

Figure 1: Cumulated Net Return of Optimized Carry Trade



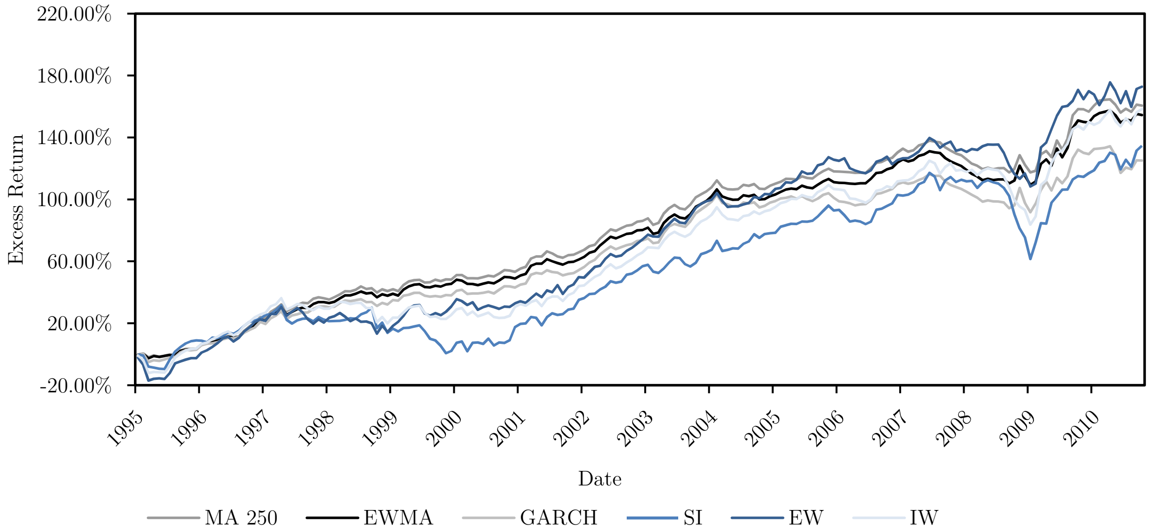
The figure plots the cumulated net return for the optimized currency carry trade with a target excess return of 6.5% in the period from December 31 1994 to October 31 2010. The optimized portfolio is rebalanced monthly. The return is the net of transaction costs.

All the strategies generated significant excess returns over the past 16 years, although the profit in 2008 was negative for all strategies. None of the strategies is significantly better than another, but the simple benchmark strategy is, as expected, the most volatile, showing a huge loss of 56% from July 2007- January 2009. The results show that the carry trade strategy is generally successful, although not every year, and some years have been quite damaging for certain strategies.

The return figures are separated into FX returns (the return from exchange rate movements $= \sum_{i=1}^{n} w_t^i \left(\frac{S_{t+1}^i}{S_t^i}\right) - 1$), IR returns (the return from the interest rates), the carry effect $(= \left(1 - \sum_{i=1}^{n} w_t^i\right) r_t^d + \sum_{i=1}^{n} w_t^i r_t^i)$, transaction costs, and the risk-free return, r_t^d. The net return is the total return less transaction costs, and the excess return is the net return less the risk-free return.

Table 3 presents the return and risk figures of these strategies. All data have been measured monthly, and are presented in an annualized form.

Figure 2: Cumulated Excess Return of Optimized Carry Trade and Benchmark Strategies



This figure plots the cumulated excess return for different currency strategies with a target excess return of 6.5% for the period from December 31 1994 to October 31 2010. MA 250 is the optimized strategy, with an estimated covariance matrix based on the moving average of the previous 250 trading days. EWMA is the optimized strategy, with the estimated covariance matrix based on the exponentially moving average, with a decay factor of 0.94. GARCH is the optimized strategy, with an estimated covariance matrix based on a DCC GARCH model. SI is the simple carry trade strategy, which buys the currency with the highest interest rate and sells the currency with the lowest interest rate. EW is the equally weighted strategy that buys the 50% highest yielding currencies and sells the 50% lowest yielding currencies. IW is the interest-weighted strategy that buys all currencies with above-average interest rates and sells all currencies with below-average interest rates, weighting them proportionally to their interest rate levels. The portfolio is rebalanced monthly. The excess return is the net of the transaction costs and the risk free rate.

The most striking result is the failure of the uncovered interest rate parity, which is disclosed by the positive FX return. According to UIP, the FX return should be the opposite of the IR return minus the risk-free return, while the excess return should be zero. However, the FX return is slightly positive and, in the case of MA 250 and EWMA, it is even significantly positive at a 10% significance level. This is an indication for the random walk theory, and for the persistence of the forward premium puzzle. Although high-yielding currencies, such as AUD, NZD, NOK, and GBP, depreciated around 20% against the USD, and the low-yielding currency JPY appreciated over 20% in 2008, the forward anomaly persists. The SI benchmark strategy had a loss of -37% in 2008, but even here the average annual FX return is positive. This means the drawdown in 2008 was only a correction of previous exaggerations. Thus, there is still no evidence for the peso theory. Furthermore, and most importantly, the excess return is significantly positive at 1% level for all strategies. This means, no matter how the carry trade was implemented, it was historically very successful.

Table 3: Risk and Return Figures of Optimized Carry Trade and Benchmark Strategies

	Optimized (MA 250)	Optimized (EWMA)	Optimized (GARCH)	Benchmark (SI)	Benchmark (EW)	Benchmark (IW)
FX Return	4.28%	3.89%	2.14%	2.39%	4.88%	3.85%
IR Return	10.29%	10.29%	10.29%	10.29%	10.29%	10.29%
Transaction costs	0.66%	0.65%	0.75%	0.40%	0.49%	0.36%
Net Return	13.93%	13.55%	11.69%	12.29%	14.70%	13.79%
Risk-free Return	3.79%	3.79%	3.79%	3.79%	3.79%	3.79%
Excess Return	10.14%	9.76%	7.90%	8.50%	10.91%	10.00%
Volatility	9.32%	9.16%	9.86%	13.76%	13.80%	12.26%
Sharpe Ratio	1.09	1.07	0.80	0.62	0.79	0.82
Best Year	34.35%	34.52%	31.52%	41.46%	53.15%	56.85%
Worst Year	-5.83%	-6.43%	-8.96%	-37.27%	-15.62%	-25.88%
FX Return <>0	*[14]	*	-	-	-	-
Net Return >0	***[15]	***	***	***	***	***
Skewness Excess Return	1.11	0.94	-0.05	-0.57	0.51	0.40

This table presents the risk and return figures for the return of different currency strategies, with a target excess return of 6.5% during the period from December 31 1994 to October 31 2010.All data are annualized, based on monthly measurements. The portfolio is rebalanced monthly. FX Return is the return that would result from the strategy if gains or losses from the interest rate differentials were to be ignored. IR return is the return that would result from the strategy if gains or losses from the exchange rate returns were to be ignored. The excess return is the net of the transaction costs and the risk-free rate.

Volatility is the most decisive factor of the results, as all the strategies were given the same target excess return of 6.5%. With the random walk assumption, the difference in the FX return is merely random. The volatility of the optimized strategies is between 9.16% and 9.86%, and is therefore substantially lower than that of the benchmark strategies, which have a volatility of between 12.26% and 13.80%. The lowest volatility has the optimized strategy EWMA, which is 9.16%. This compares to the lowest volatility of the benchmark strategies, which has an IW of 12.26%. At first glance, it might seem surprising that the EW benchmark has the highest volatility and not the SI benchmark, as the SI benchmark is less diversified.

[14] *=10% Alpha

[15] ***=1% Alpha

This surprise vanishes if no leverage is allowed[16]. The SI strategy has the most negative results in the worst year, with -37.27%, while the optimized strategies have the least worse results in the worst year, which confirms the expectations. Furthermore, the SI strategy is negatively skewed, which is considered to be an unpleasant attribute by investors. The optimized strategies are neither negatively nor positively skewed.

With regard to the Sharpe ratio, the SI benchmark has the lowest Sharpe ratio (0.62), although this is still higher than the typical Sharpe ratios of equity indices, which are below 0.5[17]. The Sharpe ratio improves further with the optimized strategies: MA 250 has a Sharpe ratio of 1.09, EWMA 1.07, and GARCH of 0.80. The high Sharpe ratios are strong evidence that the interest rate differentials cannot be explained via risk premiums. While MA 250 and EWMA reveal similar returns, the GARCH produces substantially worse return results. This can be explained by the lower FX return, while the volatility estimates seem to be similar. A measure of analyzing the forecast accuracy of the covariance estimation is the mean absolute percentage error, which compares the estimated volatility with the realized one.[18] This error is similar for all methods, being 72.4% for MA 250, 73.7% for EWMA, and 72.8% for GARCH.

Figure 3 shows the monthly values for the volatility of the AUD/USD exchange rate. GARCH, EWMA, and MA 250 are estimated values using the corresponding model, while "Realized" is the actual, realized volatility, measuring the daily volatility of the forecast month.

All methods have their advantages. GARCH has the most volatile values, similar to the realized volatility, but lagging by one month. The problem with this GARCH method is that the forecast is actually for one day, but is extrapolated for the entire month in this model. However, the results do not improve when using monthly changes as input instead of daily ones. The disadvantage of monthly values is that the change to the exchange rate can be close to zero in a month, although it was very volatile during the month. MA 250 is even more lagging than the realized volatility, but it has the advantage of being more stable than is GARCH. Thus, although MA 250 is not exactly right, on average, it is a good indicator. EWMA is less lagging than is MA 250, since the more recent values are higher weighted, but it is still close to the values of MA 250.

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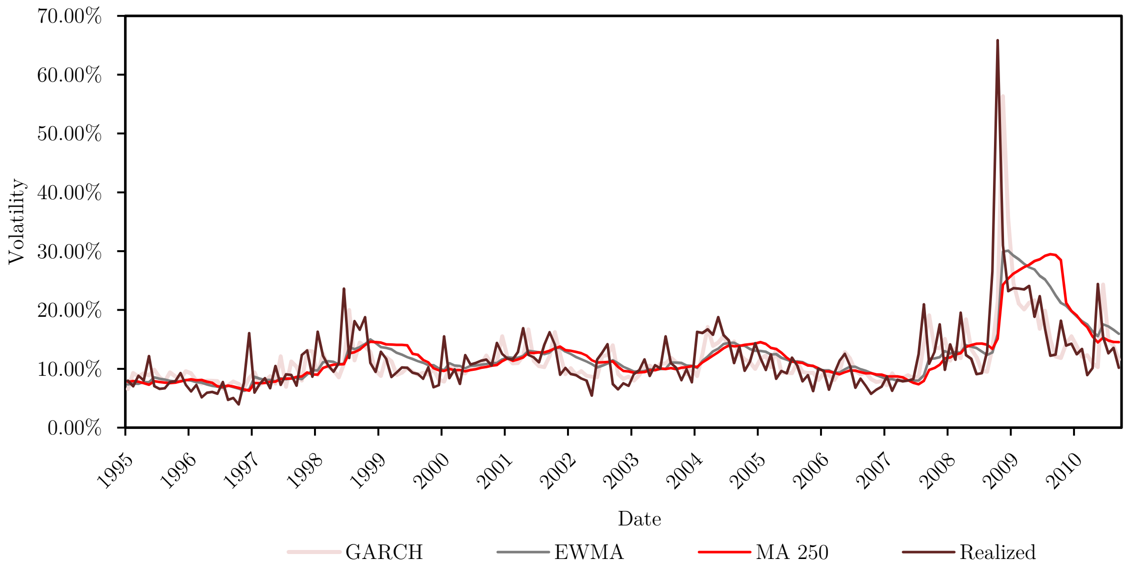
[16] With no leverage, SI has the highest return and the highest volatility, while EW has the lowest volatility and return, in line with the theory. EW has the highest Sharpe ratio (0.76), followed by IW (0.70), and SI (0.49).

[17] Fama and French [2002] evaluated the Sharpe ratio for the S&P 500, which was 0.23 over the time horizon from 1872-1950, and 0.44 from 1951-2000. The Sharpe ratio for the time horizon from 1995-2010 is 0.25 for MSCI World in USD.

[18] Russell and Adam [1987]

Figure 3: Different Volatility Estimations for the AUD/USD Exchange Rate



This figure presents the volatility estimated for the optimized portfolio with a target excess return of 6.5%, based on different covariance estimation methods, and compares it with the realized volatility using the daily-realized returns of the forecast month. The period is from December 31 1994 to October 31 2010. The portfolio is rebalanced monthly.

The transaction costs are higher in the optimized strategies, indicating that the turnover in those strategies is higher. A possible explanation is that the interest rates are very stable, while the correlation and the volatility change faster.

The IR return is 10.29% for all strategies, as implied by the risk-free rate of 3.79% and the given expected return of 6.5%.

Overall, the optimized strategy worked quite well, and the method of the covariance estimation did not make much difference. Consequently, I focus on the optimized strategy in the remainder of the paper, and I chose the MA 250 method to estimate the covariance, since the other two methods did not work significantly better than did the MA 250, which is also most easily implementable.

I.4.6 Detailed Analysis of the Optimized Strategy

Table 4 presents the risk and return figures of the optimized strategy (MA 250), again separated into four periods, each with a length of four years.

The separation offers new insights. The first three periods were very similar. The FX Return was clearly positive and, consequently, so was the excess return. The low volatility environment produced much higher Sharpe ratios, between 1.49 and 2.00. However, the skewness was around zero. In the last period of the financial crisis, the figures changed. The FX

Return was considerably lower than it was in the previous periods, but it was still positive. Transaction costs doubled with regard to the previous two periods, while the volatility increased more than twice. The higher volatility and the lower return depressed the Sharpe ratio to 0.52. However, that is still a sizeable result for the worst crisis for decades. Most astonishingly, the skewness of the net profit is positive. The portfolio composition and the leverage help to explain this result.

Table 4: Periodical Analysis of Risk and Return Figures

	1995-2010	1995-1998	1999-2002	2003-2006	2007-2010
FX Return	4.28%	4.45%	5.25%	4.97%	2.36%
IR Return	10.29%	12.18%	10.88%	9.26%	8.78%
Transaction costs	0.66%	0.76%	0.45%	0.45%	0.99%
Net Return	13.93%	15.88%	15.70%	13.80%	10.17%
Risk-free Return	3.79%	5.68%	4.38%	2.76%	2.28%
Excess Return	10.14%	10.21%	11.33%	11.05%	7.89%
Volatility	9.32%	6.04%	5.67%	7.43%	15.32%
Sharpe Ratio	1.09	1.69	2.00	1.49	0.52
Skewness Excess Return	1.11	-0.61	0.34	-0.22	1.24

This table presents the risk and return figures for the return of the optimized currency strategy, with a target excess return of 6.5% for various sub-periods for the period from December 31 1994 to October 31 2010. All the data are annualized, based on monthly measurements. The portfolio is rebalanced monthly. The FX Return is the return that would result from the strategy if the gains or losses from the interest rate differentials were to be ignored. IR return is the return that would result from the strategy if the gains or losses from the exchange rate returns were to be ignored.

Table 5 presents the best and worst monthly returns during the entire period. The worst month was December 2008, followed by the fourth largest loss in January 2009, just before the recovery of the carry trade. The second and third largest losses occurred in July 2009 and June 2010, and were mainly driven by a high leverage in these months. The best month was September 2009, followed by March 2009, June 2009 and, in fourth place, November 2008 - right in the middle of the crisis. The gain in November 2008 was driven by huge short positions in CHF, SGD, and SEK, whilst the USD was very strong against these and other currencies in this month.

Table 5: Best and Worst Monthly Net Returns

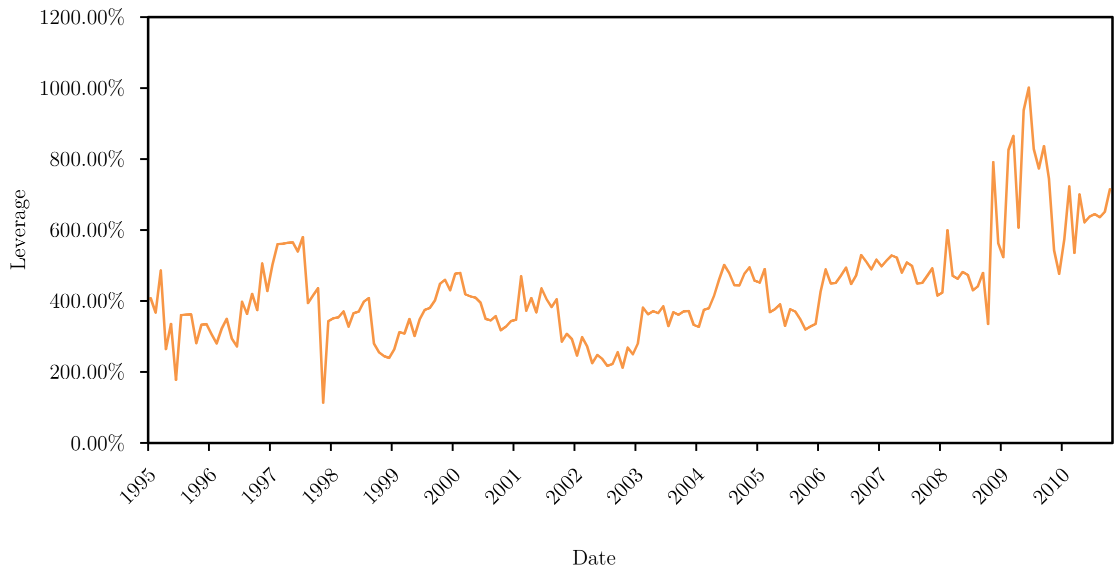
	Min	Max
1.	-6.00%	15.12%
2.	-5.79%	10.73%
3.	-5.15%	10.69%
4.	-4.87%	9.12%
5.	-4.17%	7.12%
6.	-4.10%	6.27%

This table presents the six best and worst monthly net returns for the optimized currency, with a target excess return of 6.5% during the period from December 31 1994 to October 31 2010. The portfolio is rebalanced monthly.

I.4.7 Leverage

Figure 4 shows the portfolio leverage throughout the backtest period.

Figure 4: Portfolio Leverage



This figure plots the leverage of the optimized currency portfolio, with a target excess return of 6.5% during the period from December 31 1994 to October 31 2010. The portfolio is rebalanced monthly.

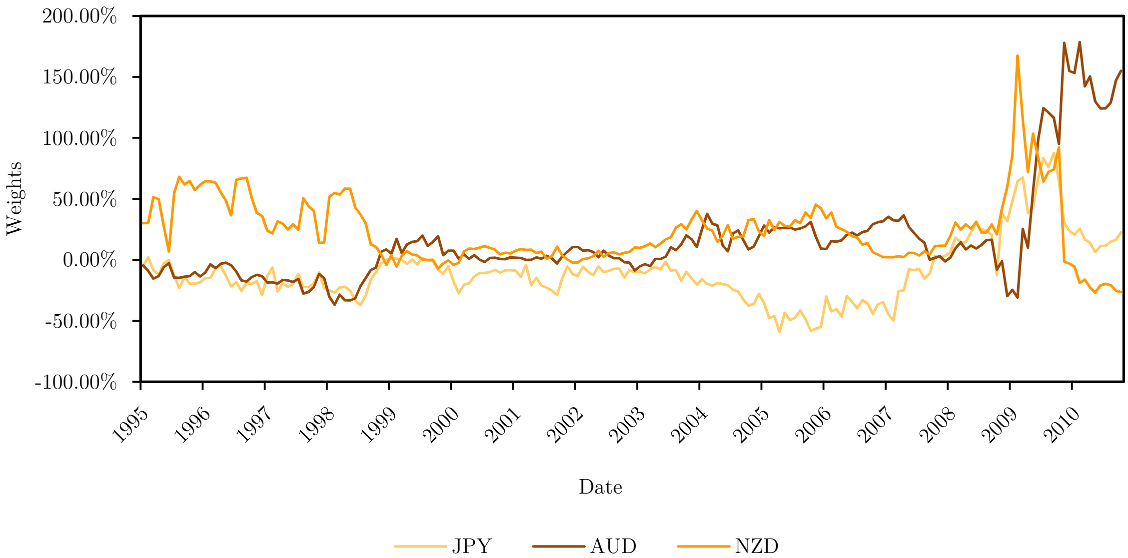
A leverage of 100% means that, for one unit of cash (in this paper, USD), the same amount is invested in foreign currencies. There are two factors that can increase the leverage, given a constant expected return. Firstly, the leverage will increase if the interest rate differentials decrease. Secondly, if the volatility of two currencies with a low interest differential decreases, more weight will be put on these currencies, which again increases the leverage. Throughout the period, the leverage was between 200% and 600%, until the financial crisis. During the crisis, the leverage increased sharply. The first increase was in November 2008, after the Lehman failure. This increase was mainly driven by shifts in volatility and correlation, since the interest rates did not change significantly and the highest difference was still 5.9%. During the following months, the exposure increased further, and spiked in May 2009 at 1001%, which was mainly driven by the converging interest rates. The largest interest differential was only 3.1% in May 2009.

The consequence for the performance of the currency portfolio was positive. The comeback for the carry trade started in February 2009, which was the first positive month for the simple carry trade after eight consecutively negative months. While the optimized strategy had to endure losses during the crisis, the gains during the rebound after the crisis more than offset the losses, since the leverage was much higher. These strong, positive results during the rebound finally led to the positive skewness in this period.

I.4.8 Portfolio Composition

Looking at the portfolio composition, there are some figures that might help to explain the superiority of the optimized results. Figure 5 plots the weights of the most famous carry trade currencies, JPY as a funding currency, and AUD and NZD as lending currencies.

Figure 5: Portfolio Weight of Selected Currencies



This figure plots the weights of selected currencies for the optimized currency portfolio, with a target excess return of 6.5% during the period from December 31 1994 to October 31 2010. The portfolio is rebalanced monthly.

Historically, the typical carry trader bought the NZD, which was the highest yielding currency, with an average interest rate of 6.5%, and sold the JPY, the lowest yielding currency, with an average of 0.1%. In the optimized currency portfolio, those currencies play different roles in the portfolio composition. JPY was not a funding currency during the recent financial crisis, even though it had the lowest interest rates. Since October 2008, the weight of the JPY has been consistently positive. The weight of the NZD and the AUD was not always positive, although a typical carry trader would have always overweighed them. In the optimized portfolio, one of the two was always positive since, most of the time, they were the highest yielding currencies. However, the other currency sometimes served as the funding currency. Why was that? A carry trade going long AUD and short JPY seems to be much more risky than one being long AUD and short NZD.[19] The short position in one of the two can be interpreted as a protection against losses in the other currencies, which is costly in the sense of lower interest rate differentials. This could be seen perfectly in November 2009, when the weight of the NZD was reduced by 94% and became negative afterwards, while the weight of the AUD increased by 83% and subsequently became even more positive. In the last year, the interest rate for AUD was 1 to 1.5% higher than for NZD, and this carry trade was exploited by the optimizer. Thus, rather than making overcrowded bets in the best-yielding currency pair, the optimizer chose positive-yielding but low-risk currencies.

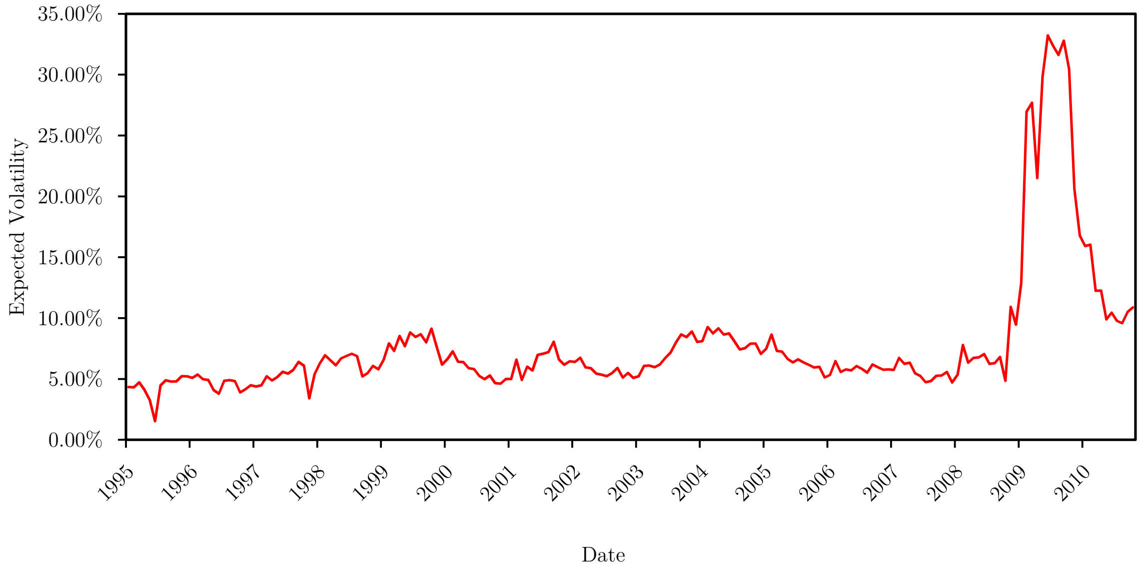
The highest average weight was not for NZD, which had an average weight of 23%, but for NOK, with 41%. Thus, long NOK seems to have been a low-risk long position. The lowest average weight was for CHF, with a weight of -69%, while the average weight for JPY was -10%. Even CAD had a lower average weight of -23%, although this is also a typical long position for carry trades. The problem of typical carry trading currencies is that they made sudden adverse moves during a crisis, caused by all carry traders unwinding their positions in order to reduce their risk. With the optimization approach, this crash risk can simply be diversified away.

I.4.9 Volatility and Correlation with Other Market Risks

The increased leverage had strong implications for the expected volatility of the portfolio. Figure 6 presents the development of the expected volatility. The expected volatility was stable at between 4% and 9% until the financial crisis. During the crisis, the volatility increased more than six-fold, up to 33.2%. There were two reasons for the increase. One was the higher leverage during the last couple of years, remembering that the leverage doubled during this period. The other reason was the higher volatility of the currencies, which tripled during the crisis.

[19] An explanation for this circumstance is that many investors that play the carry trade, and are long AUD and NZD, and short JPY. During the crisis, the return-to-risk ratio was less favorable, and many investors chose to unwind their positions when selling AUD and NZD. However, if one is long AUD and short NZD, there is no risk from such a sudden unwinding. Autumn of 2008 is a very good example: AUD and NZD lost -17.8%, respectively -16.0% in that period, while JPY gained 19.8%. A carry trade between AUD and JPY was much more risky than was one between AUD and NZD.

Figure 6: Expected Volatility



This figure plots the expected volatility of the optimized currency portfolio, with a target excess return of 6.5% during the period from December 31 1994 to October 31 2010. The portfolio is rebalanced monthly.

The volatility of the currency portfolio itself is not the only important factor. A currency portfolio is not only an own-asset class, but is always concomitant with every asset class. As an example, the MSCI World Index contains the equity component currency risk of every country listed in the index. The correlation between these market risks and the currency portfolios assists in understanding whether a reallocation of the asset classes' currency structure reduces the overall risk.

Table 6 presents the correlation of the monthly excess returns between other market risks and the currency portfolio. For the market risks, I used the equity risk (MSCI World Excess Return Index), the commodity risk (GSCI Excess Return Index), and the interest rate risk (Citigroup Global Government Bond Index). Since the carry trade strategies generate positive returns, a low or negative correlation means that a currency optimization of a distinct asset lowers the overall risk, but increases the return.[20]

The correlation analysis shows that the return of the currency portfolio is almost uncorrelated with other market risks, which means that the returns are almost independent. Although the positive correlation to equities is not a pleasant feature, the correlation is much lower (0.09) than is the correlation between commodities and equities (0.29), and between bonds and equities (0.15). This means that the currency portfolio can be added to these asset classes, and can improve the return-to-risk ratio of the combined portfolio.

[20] The return increases since the currency excess return can be added to the return of the other assets.

Table 6: Correlation between the Currency Portfolio and Other Market Risks

	Equity	Bonds	Commodity	Carry
Equity	1.00	0.15	0.29	0.09
Bonds		1.00	0.15	-0.07
Commodity			1.00	-0.05
Carry				1.00
Excess Return	3.90%	3.01%	3.38%	10.14%
Volatility	15.88%	7.13%	23.12%	9.32%
Sharpe Ratio	0.25	0.42	0.15	1.09

This table presents the correlation between various market risks and the optimized currency portfolio, with a target excess return of 6.5% during the period from December 31 1994 to October 31 2010. The portfolio is rebalanced monthly. All data are annualized, based on monthly measurements.

A glance at the individual asset classes reveals that the currency portfolio was the most successful during the backtest period, clearly having the highest return and Sharpe ratio. The equities had a historically low excess return of only 3.9% during this period, influenced by two worldwide recessions in 2001/2002 and in 2008/2009, which lead to a low Sharpe ratio of 0.25. Bonds had quite a good environment for positive excess returns, since interest rates were in a downward trend during the entire period, generating a Sharpe ratio of 0.42. Commodities had the lowest Sharpe ratio, because they were been extremely volatile, with 23.12% volatility per annum. The currency portfolio is not only the best asset from a single asset perspective, but is also the most favorable combination with other assets.

Bliss and Panigirtzoglou [2004], as well as many others, showed that investors are generally risk averse. In the event that returns are skewed, the risk aversion increases (compare this with Harvey and Siddique [2000]). Thus, the negative skewness of the simple carry trade, accompanied by some returns that were significantly worse and the positive correlation with equity and commodity indices, which was most pronounced during the recent financial crisis, might be the main reason why some investors abandoned the carry trades. Nonetheless, the forward premium puzzle remains, since the optimized currency portfolio is not negatively skewed, has a better Sharpe ratio, and the correlation with other market risks is almost inexistent. During the past 16 years, an investment in an optimized currency portfolio was low risk and was profitable.

I.4.10 Inter-temporal Weighting Scheme

Investors have different requirements for inter-temporal returns. Some want to achieve a constant return, which is what the optimization (5) intends to achieve. Others want to have a constant volatility, which could also be achieved by adding the constraint $\sqrt{w_t' \Sigma_t w_t} = \sigma$ in (5). The math can be simplified by using the results of (5) to calculate the expected volatility. The weights can then be recalibrated proportionally, so that the expected volatility reaches the desired level. With this closed-end formula, no optimization is necessary. A third possibility for inter-temporal returns is to have a constant exposure, which means that the sum of the absolute values of the weights is constant. If the sum is 1 and the basis is 1 USD, then there is no leverage. Again, these weights can be calculated using formula (5), by dividing the resulting weights by the sum of the absolute values of all weights.[21]

The constant return strategy ensures, as the name says, a constant expected return. The method using constant volatility has the advantage that the exposure will be reduced when the expected volatility increases. From an inter-temporal perspective, bets are higher when the risk is lower, and vice versa. The constant exposure strategy is often a given constraint on the part of the regulator, who does not allow leverage for many investors. Table 7 compares the results of the different weighting strategies.

Table 7: Risk and Return Figures of Various Inter-temporal Weighting Strategies

	Constant Return (6.5%)	Constant Volatility (16%)	Constant Exposure (100%)
FX Return	4.28%	7.69%	0.87%
IR Return	10.29%	20.34%	5.38%
Transaction Costs	0.66%	1.48%	0.14%
Net Return	13.93%	26.58%	6.11%
Risk-free Return	3.79%	3.79%	3.79%
Excess Return	10.14%	22.79%	2.32%
Volatility	9.32%	18.15%	1.76%
Sharpe ratio	1.09	1.26	1.32
Skewness Excess Return	1.11	-0.01	-0.06

This table presents the risk and return figures for various weighting strategies of the optimized currency portfolio. The second column has a target excess return of 6.5%. The third column has a target volatility of 16%, and the last column scales the weights of a target excess return of 6.5% down, so that there is no leverage. The period is from December 31 1994 to October 31 2010. The portfolio is rebalanced monthly. All data are annualized, based on monthly measurements.

A constant expected excess return of 6.5% does not guarantee that the realized excess return is equal (10.14%), nor does a constant expected volatility of 16% guarantee that the realized volatility is equal (18.15%). The first phenomenon can be explained by the uncertainty of FX

[21] Individual constraints for every currency can also be added to formula (5). In this case, the Sharpe ratio maximization formula is more suitable for guaranteeing the maximal Sharpe ratio. Risk minimization at a given return only maximizes the Sharpe ratio if there are no individual constraints. I suggest using the equations with MATLAB, employing the barrier method algorithm provided by Knitro 6.0.

returns, while the latter can be explained by the volatility estimation error. The volatility over time is higher than is the expected volatility, because the uncertainty in the realized volatility (the volatility of the deviations in the volatility) increases the average realized volatility. For example, if the volatility was 12% in period 1 and 20% in period 2, the average of periods 1 and 2 would be 16%, but the volatility measured over both periods is higher than 16%, due to the quadratic error measurement.

The constant volatility and the exposure strategy both have higher Sharpe ratios than does the constant return. The relatively low Sharpe ratio of the constant return strategy is caused by the greater inter-temporal volatility of the returns. While the volatility tripled during the financial crisis, the expected return sank by only about 50%. The volatility was more "volatile" than was the return; as a result, stabilizing the volatility was more successful for the Sharpe ratio. The constant exposure strategy is somewhere between constant return and constant volatility. Apparently, this well-balanced method produced the best Sharpe ratio.

The skewness of the constant volatility and exposure strategy is slightly negative. As we have seen, the positive skewness in the constant return strategy was produced by very high weights at the end of the crisis at the beginning of 2009. These returns were significantly higher than any previous returns, and thus pushed the skewness into the positive area, although it was negative before 2007. The negative skewness of the latter two strategies gives a better image of the reality for normal days, since the positive in the constant strategy was shaped by a single event.

I.4.11 Interest Rates versus Forward Rates

It does not matter whether I used interest rates or forward rates, if the covered interest rate parity holds. Historical evidence shows that the covered interest rate parity holds often, but not always. In this dataset, there have been two major violations of CIP. There has been a long-term violation in SGD, and a violation of all currencies during the months of the Lehman failure, when the markets were in dysfunction.

Table 8 presents the results of forward rates again, this time using Bloomberg's data compared to interest rates from Datastream, once in exactly the same time period and once in the full time period available, which is a further eight years. The latter cannot be compared directly with the others, but helps one to see that the carry trade is also profitable over a longer time horizon. Table 8 reveals that returns using forward rates differ from the returns using interest rates, even if the time period is exactly the same. There are two reasons for the different returns: in those cases where CIP does not hold, the different return as an input for the optimization produces different optimal weights. Furthermore, the different interest return obviously changes the return itself. These two factors can be separated as follows:

If the weights produced by the forward rate optimization are multiplied by the realized return using interest rates, the IR return is 10.17% instead of 10.29%. This substitution produces a small return difference, while the volatility remains largely unchanged, since the weights are still the same. This means that it does not really matter which interest rates are used to calculate the portfolio return. If the weights produced by the interest rate optimization are multiplied by the realized return using forward rates, the IR return is 9.99%, and the FX return is 2.56% instead of 4.28%. Once again, the IR return does not change much. The FX return difference is more affected, since the returns are volatile. However, the difference in FX return is random under the random walk hypothesis. Thus, it does not really matter whether forward rates or interest rates are used for the IR return, although the total return is different because of different weights that produce different FX returns.The main difference between the two is the different SGD interest rates. The average forward implied interest rate is 2.4%, whereas the interbank interest rate average is 1.9%. Consequently, the average SGD weight in the optimization with forward rates was -56%, while it was -88% in the optimization with interest rates.

The backtest with interest rates starting in 1987 has an excess return of 6.69%, which is very close to the target return of 6.5%. Thus, over a longer time horizon, the FX return is very close to a random walk, with only +0.71%. The Sharpe ratio is lower, especially because the FX return is lower.

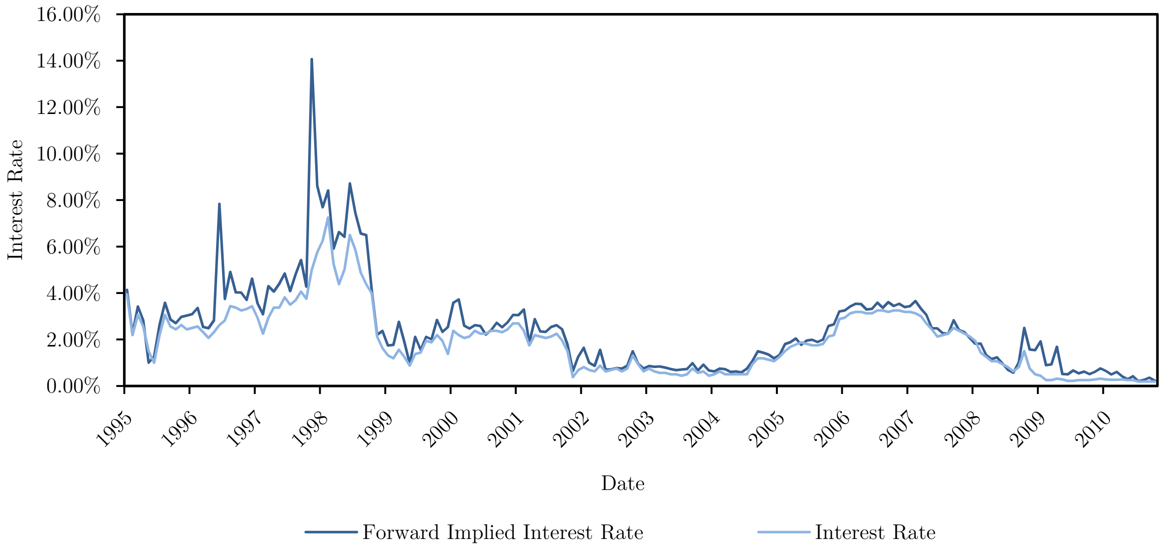
Table 8: Return of Interest Rates and Forward Rates

	Forward Rates (1995-2010)	Interest Rates (1995-2010)	Interest Rates (1987-2010)
FX Return	4.28%	2.61%	0.71%
IR Return	10.29%	10.29%	11.11%
Transaction Costs	0.66%	0.50%	0.50%
Net Return	13.93%	12.42%	11.30%
Risk-free Return	3.79%	3.79%	4.61%
Excess Return	10.14%	8.63%F	6.69%
Volatility	9.32%	8.50%	7.56%
Sharpe Ratio	1.09	1.02	0.89
FX Return <>0	*	-	-
Skewness net Return	1.02	0.25	-0.11

This table compares the risk and return figures for interest rate and forward rates for the optimized portfolio, with a target excess return of 6.5%. The period is from December 31 1994 to October 31 2010 for columns two and three, and from December 31 1986 to October 31 2010 for the last column. The portfolio is rebalanced monthly. All data are annualized, based on monthly measurements.

Figure 7 shows the historical SGD LIBOR and the forward implied interest rate of SGD, implied by the SGD/USD forward rate and the USD LIBOR. The forward implied interest rate was often higher than was the LIBOR and, in 1997, it was more than twice as high, which is a clear violation of CIP. The rates have been closer together since 2002 but, during the financial crisis, there again was a deviation. The violation of CIP is caused by capital constraints for foreign investors. Before 2002, a trade had to be related to an economic activity (Hohensee and Lee [2004]).

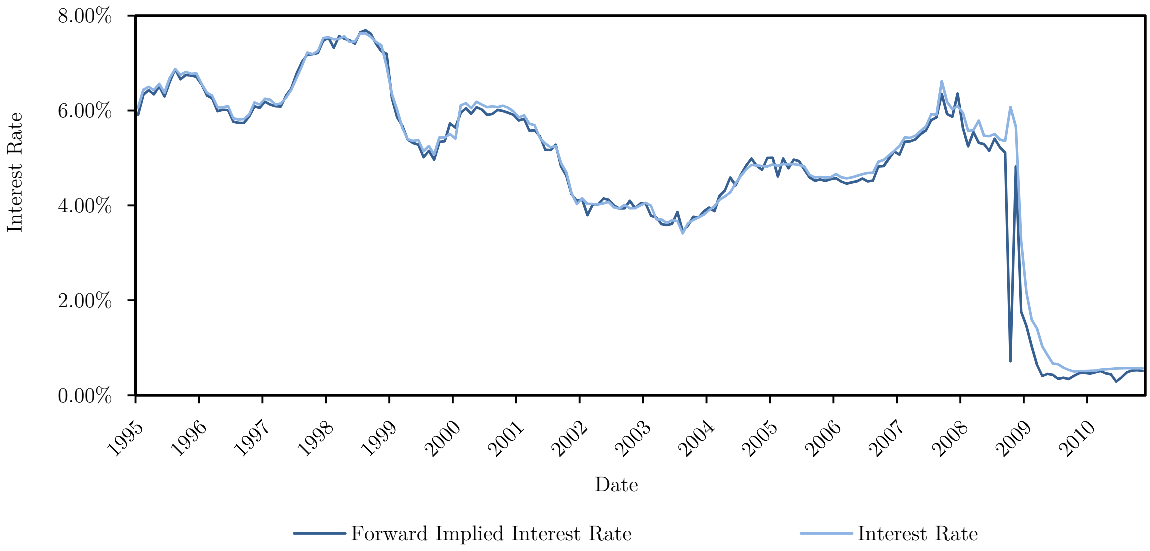
Figure 7: Forward Implied Interest Rate versus LIBOR Rate SGD



This figure compares the LIBOR interest rate for SGD with the SGD interest implied by the SGD-USD forward rate. The period is from December 31 1994 to October 31 2010.

Figure 8 presents the interest rates for a fully free-tradable currency; here, GBP is used as an example. The forward implied rate tracked the LIBOR very closely until the financial crisis, when the forward rate was below that of the LIBOR. During the crisis, arbitrage was more difficult, since the volatility increased and the risk limits for arbitrageurs were cut. Furthermore, there was an immense need for USD since many products had to refinance with USD. The highest difference was 5.4% in September 2008, the month of the Lehman failure.

Figure 8: Forward Implied Interest Rate versus LIBOR Rate GBP



This figure compares the LIBOR interest rate for GBP with the GBP interest implied by the GBP-USD forward rate. The period is from December 31 1994 to October 31 2010.

I.4.12 The Special Case of the Danish Krona

The currency DKK is excluded in the studies above and, together with HKD, is the only currency in the developed markets that was excluded. The reason for this measurement is that the DKK is pegged to the EUR in a small range, while all the other currencies are free-floating[22]. The volatility of the EUR/DKK exchange rate was only 1.2%. Including DKK partially distorts the results of the other currencies. The effect is presented in Table 9.

The inclusion of the DKK boosts the Sharpe ratio to 1.27. However, in this case it is actually a peso problem. The pegging of the DKK to the EUR did work during these 16 years, but it is not guaranteed that it will work forever. Figure 9 shows that the weight of the EUR and the DKK was often contrary, as one was positive while the other was negative, depending on which had the higher interest rate. There was a fear during the financial crisis that Denmark could collapse, which pushed interest rates for the DKK more than 1% above the EUR rates. This interest rate difference is exploited by the optimization, since it is nearly risk-free, based on the historical volatility. However, the probability of a future collapse is not considered in the optimization. Since the DKK is pegged to the EUR, the risk will not appear in the exchange rate and thus neither in the optimization, which can be regarded as a lack of a backward-looking volatility model.

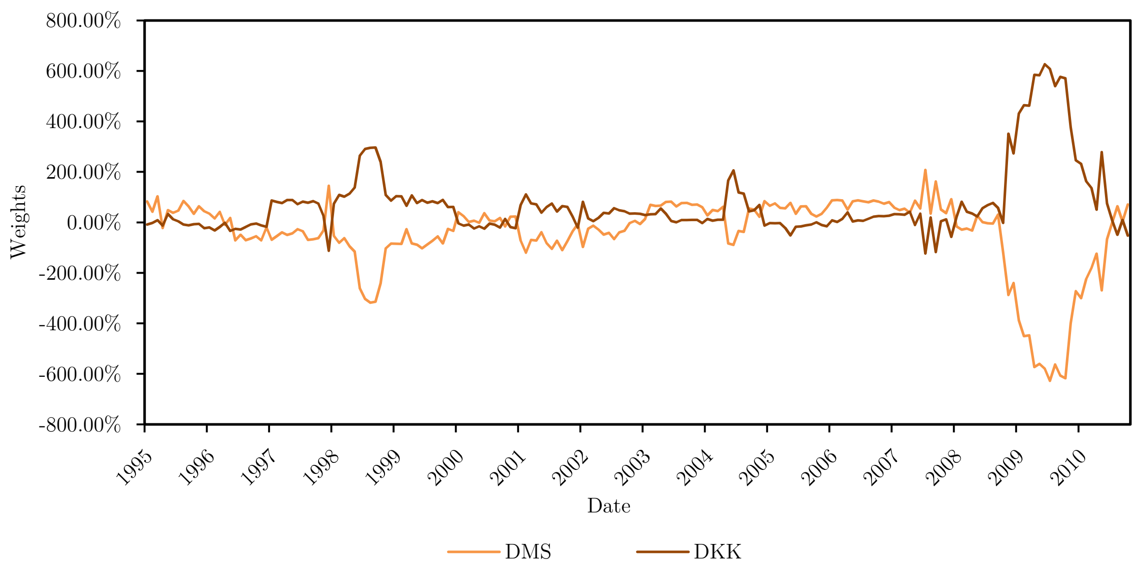
[22] Except for the second excluded currency, the HKD, which is pegged to the USD.

Table 9: Inclusive of the DKK Currency

	Excluding DKK	Including DKK
FX Return	4.28%	3.06%
IR Return	10.29%	10.29%
Transaction Costs	0.66%	0.76%
Net Return	13.93%	12.61%
Risk-free Return	3.79%	3.79%
Excess Return	10.14%	8.82%
Volatility	9.32%	6.95%
Sharpe Ratio	1.09	1.27
Skewness Net Return	1.02	-0.19

This table compares the risk and return figures for the optimized portfolio, with a target excess return of 6.5%, with and without the currency DKK. The period is from December 31 1994 to October 31 2010. The portfolio is rebalanced monthly. All data are annualized, based on monthly measurements.

Figure 9: Historical Weight of the DKK and the EUR



This figure plots the weights of the DKK and the DMS for the optimized currency portfolio, with a target excess return of 6.5%, for the currency universe including DKK. The period is from December 31 1994 to October 31 2010. The portfolio is rebalanced monthly.

I.4.13 Extension of the Currency Universe

The currency universe in the previous empirical analysis was chosen according to the data available. Nonetheless, there are more data available for a shorter time horizons or for different investment horizons that prove the robustness of the results.

From December 29 2000, the currency universe could be extended to 15 currencies, with the additional currencies HUF, PLN, ZAR, and MXN. These countries are commonly known as emerging market currencies. Thus, all currencies that are tradable without restrictions[23] are included in the dataset, except for TRY, CZK, ISK, and ILS, where data are not available for the entire period. Non-deliverable forwards[24] are also excluded from the dataset, since CIP is not guaranteed for those contracts.

I divided the currency universe into three baskets:

7 currencies: USD, CHF, EUR, JPY, GBP , AUD, CAD

11 currencies: USD, CHF, EUR, JPY, GBP , AUD, CAD, NOK, SEK, SGD, NZD

15 currencies: USD, CHF, EUR, JPY, GBP , AUD, CAD, NOK, SEK, SGD, NZD, HUF, PLN, ZAR, MXN

Table 10 presents the results for optimized currency portfolios with different amounts of currencies.

Table 10: Backtest Results for Different Currency Baskets

	7 Currencies	11 Currencies	15 Currencies
FX Return	0.06%	2.16%	-0.12%
IR Return	8.94%	8.94%	8.94%
Transaction Costs	0.47%	0.73%	0.66%
Risk-free Rate	2.44%	2.44%	2.44%
Gross Return	8.56%	10.41%	8.18%
Excess Return	6.12%	7.97%	5.74%
Volatility	14.45%	11.13%	5.92%
Sharpe Ratio	0.42	0.72	0.97

This table compares the risk and return figures for the optimized portfolio, with a target excess return of 6.5% for the various currency universes. The 7-currency universe includes USD, CHF, EUR, JPY, GBP, AUD, and CAD. The 11-currency universe includes USD, CHF, EUR, JPY, GBP, AUD, CAD, NOK, SEK, SGD, and NZD. The 15-currency universe includes USD, CHF, EUR, JPY, GBP, AUD, CAD, NOK, SEK, SGD, NZD, HUF, PLN, ZAR, and MXN. The period is from December 31 2001 to October 31 2010. The portfolio is rebalanced monthly. All data are annualized, based on monthly measurements.

The bigger the currency basket, the higher the Sharpe ratio. This is consistent with the theory, since the result from an unconstrained basket should not decrease if the universe is increased,

[23] According to the ZKB trading unit, status 01.01.2009

[24] A non-deliverable forward is a forward contract for a currency that is not deliverable. At maturity, there is only a profit or loss settlement, which will be in a currency that is deliverable.

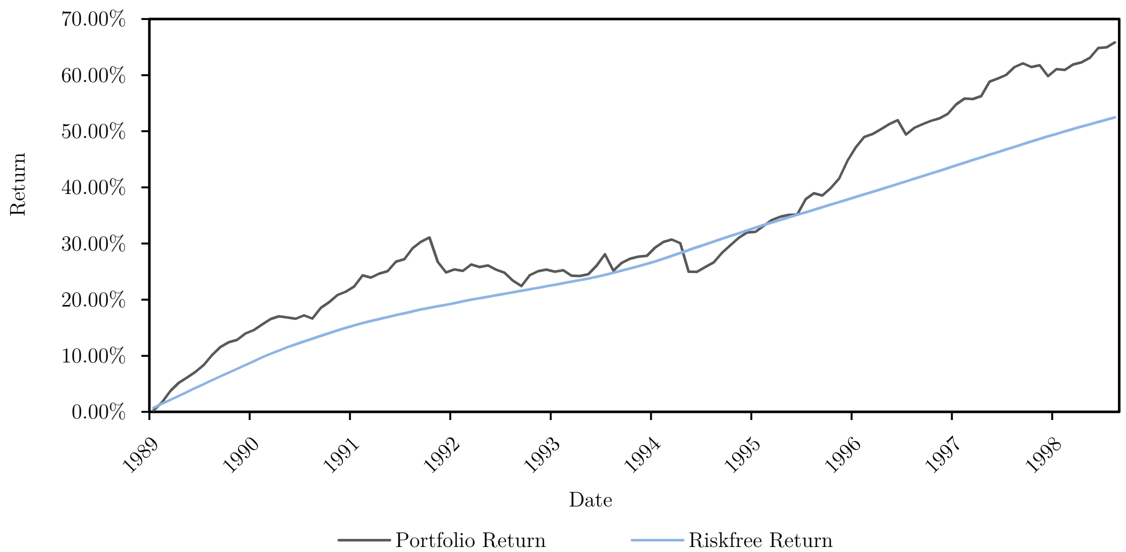
given that the volatility can be estimated sufficiently precisely. The higher Sharpe ratio is driven by the lower volatility, which profits from the improved degree of diversification.

Interestingly, the 15-currency basket is the only one that has a negative FX. This might be an indication that the additional four currencies depreciated in the direction of the UIP. This result for these so-called emerging market currencies is consistent with Burnside, Eichenbaum, and Rebelo [2007]. The average interest rate for the four currencies over the entire sample was 7.5%, while it was 2.9% for the other 11 currencies. Thus, the extreme interest rate difference caused the depreciation of the high interest rate currencies to a small extent. Nevertheless, the UIP can be rejected, as the net profit is significantly positive.

Baz, Breedon, Naik, and Peress [2001] conducted a similar analysis, using the currencies USD, CHF, EUR, JPY, and GBP. They set $\mu_0 - r_t^d = 2\%$, and used optimization (5). The methodology for the variance-covariance matrix is not precisely defined, except that they took an estimation window of two years. The interest rate source is LIBOR, and the rebalancing day is the first day of the month. The backtest period is from November 1989 to June 1999.

I recalculated their results using (6) for the variance-covariance matrix with a moving average of 500 days. Figure 10 presents the cumulated return of my calculations.

Figure 10: Cumulated Return of Baz et al.



This figure plots the cumulated excess return for the optimized currency carry trade, with a target excess return of 2% during the period from October 31 1989 to June 30 1999. The currency universe consists of USD, CHF, EUR, JPY, and GBP. The optimized portfolio is rebalanced monthly. The excess return is before transaction costs.

Figure 10 looks very similar to the results in Baz et al., yet the result is different. Baz et al. have an excess return of 2.38% and a volatility of 3.96%, while my calculations resulted in a return of 1.35% and a volatility of 4.13%. Furthermore, Baz et al. report a Sharpe ratio of 0.60, while I obtained only 0.33. The problem is that the methodology in Baz et al. is too imprecisely described to make an exact replication. Also, the source of the exchange rates is not declared.

I.4.14 Investment Horizon

The investment horizon and rebalancing frequency up to this point has been one month. Table 11 presents the results for different investment horizons. All results are annualized with aggregated data of three months, in order to obtain a comparison with the longest time horizon. This is especially important for the volatility, where I annualized the quarterly results by multiplying them by the square root of 4. The figures for the one-month horizon are slightly different from the results in Table 3, because here the backtest ends on September 30, instead of October 30, so as to match the data of the three-month horizon, and the volatility is computed with quarterly results.

Table 11: Backtest Results for Different Investment Horizons

	1 Week	1 Month	3 Months
FX Return	3.80%	4.37%	4.98%
IR Return	10.25%	10.31%	10.46%
Transaction costs	1.89%	0.66%	0.42%
Risk-free Rate	3.75%	3.81%	3.96%
Gross Return	14.10%	13.82%	13.79%
Excess Return	8.45%	9.99%	9.77%
Volatility	8.17%	8.67%	9.09%
Sharpe ratio	1.03	1.15	1.08

This table compares the risk and return figures for the optimized portfolio, with a target excess return of 6.5% for different rebalancing frequencies. The currency universe includes USD, CHF, EUR, JPY, GBP, AUD, CAD, NOK, SEK, SGD, and NZD. The period is from December 31 1994 to September 30 2010. All data are annualized, based on monthly measurements.

The analysis shows that it does not matter how long the investment horizon is. All horizons achieve Sharpe ratios above 1. The risk-free rate increases with the investment horizon, since the yield curve increased. The advantage of the shorter horizon seems to be a better and faster adaption to new circumstances, which results in lower volatility. However, transaction costs are higher for shorter horizons, due to the higher turnover. Thus, the optimization results generally improve with a shorter time horizon, but the higher transaction costs offsets the gains. If we only compare the expected excess return net of the transaction costs with the realized volatility and do not consider the random FX return, then the one-month horizon is the optimal one. In one week, transaction costs increase by 1.23% for a reduction of 0.5% volatility, which is not a good trade-off in terms of the Sharpe ratio. In three months, the transaction costs reduce by 0.24%, for a volatility increase of 0.42%, which not a good result either.

I.4.15 Constraining the Sharpe Ratio

Since investors like a high Sharpe ratio, the strategy can be extended, with a restriction of investing only in the carry trade when the expected Sharpe ratio has at least a certain level. This has the additionally attractive side effect that the carry trade will not be implemented when all the interest differentials are zero, or even when they are close to zero.

Table 12: Backtest Results for Various Expected Sharpe Ratios

	<0.8	>=0.8	>=1	>=1.2	>=1.4
Excess Return	1.93%	2.42%	2.80%	2.42%	3.90%
Volatility	2.29%	1.61%	1.63%	1.64%	1.22%
Sharpe ratio	0.84	1.50	1.71	1.47	3.21
Skewness	0.02	-0.06	-0.11	0.11	-0.04
Number	38	152	109	55	15

This table compares the risk and return figures for the optimized portfolio, with a target excess return of 6.5% for an unlevered portfolio when restricting it to a certain level of an expected Sharpe ratio. >= 0:6 means that the expected Sharpe ratio must be at least 0.6 for the following period to be considered. The realized return figures are then computed for the months in which the restriction was fulfilled. The portfolio is rebalanced monthly. The period is from December 31 1994 to October 31 2010. All data are annualized, based on monthly measurements.

Table 12 shows the results of the optimized strategy for different constraints of the expected Sharpe ratio. Here, I used the methodology of constant exposure in order to compare the market volatility.

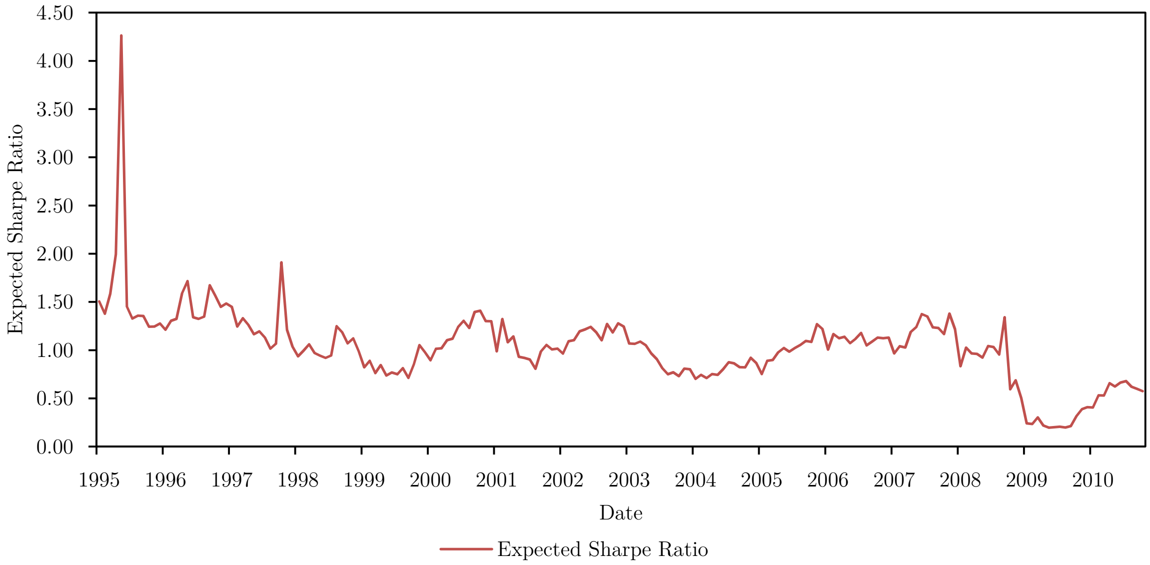
The restriction of the expected Sharpe ratio is quite effective. If the dynamic strategy is implemented only when the expected Sharpe ratio is at least 0.8, the realized Sharpe ratio is boosted to 1.50, in contrast to the situation in which there is no restriction, where the Sharpe ratio is 1.32 (Table 7). However, this requires that there was no investment in the carry trade since November 2008, when the expected Sharpe ratio dropped below 1. The expected Sharpe ratio had its lowest value on August 31 2009, with 0.20, while the last value in the sample reports an expected Sharpe ratio of 0.57. If we were to only invest if the expected Sharpe ratio were lower than 0.8, the realized Sharpe ratio would have been 0.84. Thus, the expected Sharpe ratio predicts the realized Sharpe ratio.

There are two reasons for the lower than expected Sharpe ratio. Firstly, the volatility spiked after the Lehman collapse on September 15 2008, from 1.4% in August 2008 to 3.8% in August

2009. These figures were measured for an unlevered portfolio, in order to distinguish between expected returns and expected volatility. Thus, the financial crisis is a very good example of the carry trade crash risk. However, volatility cannot be the explanation for the lower-than-expected Sharpe ratio in recent years, as the expected volatility dropped back to normal levels in May 2010 (see Figure 6). The main reason for the sustainably lower level of the expected Sharpe ratio is the convergence of global interest rates that are close to zero, a response from central banks to poor economic growth since the financial crisis. Thus, the expected return on the unlevered portfolio is only half of what it was before the crisis, and the same holds for the realized return. However, the lower return of the carry trade has not been caused by the crash itself during the Lehman collapse, as returns on the diversified carry trade were positive in September and in October 2008. Therefore, the crash risk was diversified away.

Although the carry trade has been historically a very successful strategy and will probably be so in the future, at present the carry trade is not as attractive as it was previously. In order for the carry trade to get back to the same success level that it experienced historically, the expected Sharpe ratio needs to increase to more elevated levels, which first requires that interest differentials increase to higher levels.

Figure 11: Expected Sharpe Ratio 1995-2010



This figure presents the expected Sharpe ratio for the optimized portfolio, with a target excess return of 6.5% for an unlevered portfolio. The portfolio is rebalanced monthly. The period is from December 31 1994 to October 31 2010. All data are annualized, based on monthly measurements.

I.5 Conclusion

The carry trade has traditionally had a reputation for risk. In some ways, this reputation is well earned. Undiversified portfolios face the risk of large losses, as realized in the wake of the Lehman Brothers' bankruptcy. For example, an investor who was long in the Australian dollar and short in the yen would have lost 22% in October of 2008. Even a trader who held a diversified 1/n portfolio would have lost -7.79% from July to December 2008. In sharp contrast, the diversified portfolio constructed using a mean-variance analysis had a positive performance (+0.62%) during that period. A portfolio constructed using a mean-variance analysis can identify opportunities that a more heuristic method will not detect. For example, a mean-variance analysis recognizes that, while the yen has a lower interest rate, the Singapore dollar is a more desirable investment because of its superior risk/return trade-off. Once sufficiently diversified, the carry trade is revealed to have been a surprisingly low-risk strategy over the last 20 years, and the crash risk has been diversified away via a mean-variance optimization. Furthermore, returns have a low correlation to other assets classes and have better Sharpe ratios than do equities, bonds, and commodities. The results are robust for the investment horizon, the currency universe, and the optimization methodology.

During the last couple of years, the carry trade had been less successful, as most interest differentials converged close to zero. Therefore, the expected Sharpe ratio from the carry trade strategy is well below 1, which means that investment in the carry trade is not currently as attractive as it was before, unless interest differentials increase to a more elevated level.

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Part II

Short-Horizon UIP with Long-Term Interest Rates

Short-Horizon UIP with Long-Term Interest Rates

Fabian Ackermann

May 28, 2013

Abstract

The failure of uncovered interest rate parity to explain short-term exchange rate movements is well documented. The return of a long-term bond should be equal to the risk-free rate under the expectation theory of interest rates. Given this theory, the uncovered interest rate parity should be equal for short- and long-term interest rates. However, I will show that the uncovered interest rate parity fails for short-term interest rates but holds for long-term interest rates, even over short horizons. Furthermore, controlling for a time-varying risk of the exchange rate improves the relationship with the long-term interest rates. These ambiguous results are caused by the failure of the expectation theory for the term structure of interest rates, as long-term interest rates are a bad predictor for future short-term interest rates. The significance increased over the last decade, as the liquidity of the exchange rate and the interest rate market increased. The results are robust for interest-rate maturities of between 12 months and 30 years.

II.1 Introduction

Uncovered interest rate parity (UIP) is a controversial theory that connects exchange rates and interest rates. According to UIP, the exchange rate between countries with high interest rates and countries with low interest rates should depreciate to exactly the same degree as the interest differential between the two countries.

Despite its inherent plausibility, empirical evidence provides little support for the UIP theory. Several studies have found that exchange rates behave contrary to the expectations of UIP, in that currencies with high interest rates appreciate against currencies with low interest rates.

A large quantity of literature is devoted to investigating the puzzle of UIP failure. Bacchetta [2001] summarizes the recent explanations for UIP failure, namely risk premium, limited participation, and deviations from rational expectations. Risk premium means a small deviation from UIP can be accepted, as investors demand a risk premium to push exchange rates towards UIP. The limited participation rationale says that the exchange rate is not always a fair economic value, as only a fraction of foreign currency holdings is actively managed. In the deviations from the rational expectations theory, investors either believe that interest differentials are more temporary than they really are, or investors are ambiguity-averse.

In this study, I will reconsider the question of UIP. In previous studies, UIP was typically tested via interest rates that were being kept until maturity. I used the returns on 10-year interest rate swaps over 1-month horizons to test UIP for all G7 currency pairs. The main difference is that the return of the swap consists not only of interest rate earnings, but also of changes in interest rates. Using the return of the 10-year interest rate has the advantage that future interest rate expectations, and therefore deviations from rational expectations, are also included in the regression. The difference between the expected and the unexpected return of the 10-year interest rate can be measured. Furthermore, I used the monthly change of the currency volatility to account for the fact that investors are risk-averse, and risk varies over time. The relationship becomes stronger over the sample period as the liquidity of the exchange rate market increases.

II.2 Uncovered Interest Rate Parity

II.2.1 Main Idea

The uncovered interest rate parity model (UIP)[1], first proposed by Fisher [1896], states that a currency with a high interest rate should depreciate compared to a low interest rate currency, ceteris paribus, in order to exactly offset the gains received by the higher yield:

$$\mathrm{s_{t+1}} - \mathrm{s_t} = \mathrm{i_t} - \mathrm{i_t^*}. \tag{1}$$

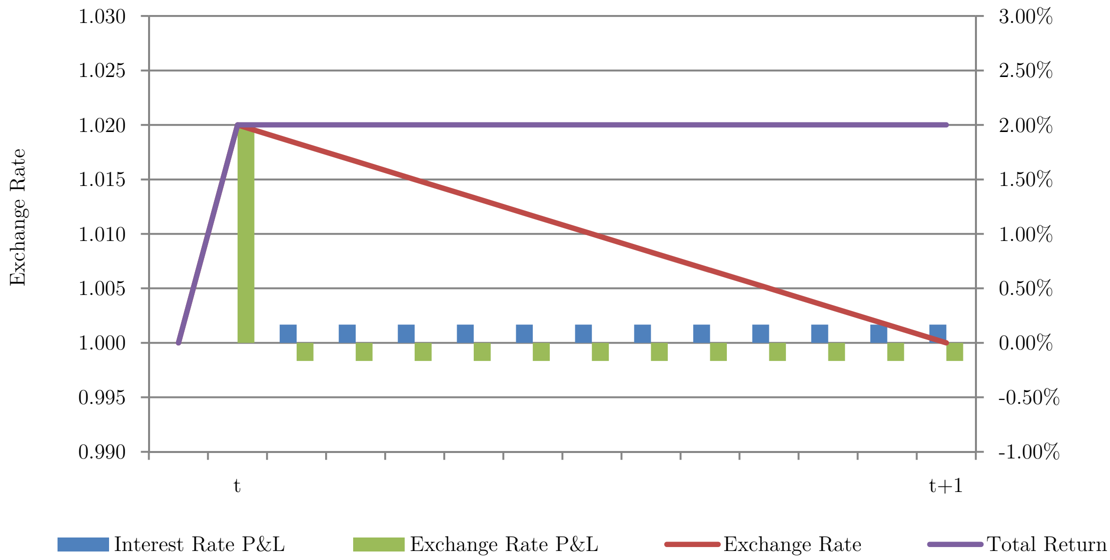
Where $\mathrm{s_{t+1}} - \mathrm{s_t}$ is the change of the log spot exchange rate, $\mathrm{s_t}$ is quoted as the amount of domestic currency necessary to buy one unit of the foreign currency, and $\mathrm{i_t} - \mathrm{i_t^*}$ is the interest differential between the domestic and the foreign interest rates. This means that if the interest differential between the domestic and the foreign currency is negative (the foreign interest rate is higher than is the domestic interest rate), $\mathrm{s_{t+1}} - \mathrm{s_t}$ is also negative and the foreign currency will depreciate by $\mathrm{i_t} - \mathrm{i_t^*}$.

To illustrate the relationship, let us assume that there is a hypothetical currency pair with an exchange rate of 1, and interest rates are equal in both currencies; consequently, the interest rate differential is 0. At time t, the central bank of the foreign country announces an increase in the interest rate of 2% for one period. After that period, the interest rate will again decrease to the same level as it is in the domestic country. Figure 1 presents the theoretical development of the exchange rate, and the total return.

At the time that the interest rate of the foreign country is increased, the exchange rate (in this example, quoted as the amount of domestic currency necessary to buy one unit of the foreign currency) will appreciate by 2%, from 1 to 1.02, since the foreign currency has a 2% higher value due to the interest rate advantage. The total return from this investment is 2% at time t. However, according to UIP, the exchange rate will again depreciate, from 1.02 to 1 between t and t+1. The exchange rate loss between t and t+1 is offset by the gain of the higher interest rate; thus, the total return will remain unchanged at 2%.

[1] UIP is also known as the international Fisher effect.

Figure 1: Exchange Rate and Total Return Development



This illustration shows the effect of an unexpected interest rate increase on the future exchange rate and the interest rate return, with the assumption of risk-neutral investors.

The standard regression to test UIP measures whether the interest gain between t and t+1 is offset by the exchange rate loss (or vice versa), using the following regression:

$$s_{t+1} - s_t = \alpha + \beta_{Level}(i_t - i_t^*) + e_{t+1} \tag{2}$$

where e_{t+1} is the disturbance term that is independent from the interest rate difference and the exchange rate. The UIP hypothesis is a test of $\alpha = 0$ and $\beta_{Level} = 1$.

Another way to test UIP is the following regression:

$$s_{t+1} - s_t = \alpha + \beta_{Level}(f_t - s_t) + e_{t+1} \tag{3}$$

where f_t is the logarithm of the forward rate, which is the 'today's value' of the future spot rate s_{t+1}. Using the covered interest rate parity, f_t is defined as follows (log rates):

$$f_t = s_t + i_t - i_t^*. \tag{4}$$

The problem with both regressions (2) and (3) is the case in which the interest differential is 0. This problem can be avoided by substituting the regressions as follows:

$$s_{t+1} - s_t - i_t^* = \alpha + \beta_{Level}(i_t) + e_{t+1}. \tag{5}$$

$$s_{t+1} = \alpha + \beta_{Level}(f_t) + e_{t+1}. \tag{6}$$

II.2.2 Empirical Evidence

Previous research provides little evidence in favor of this hypothesis. Please refer to Hodrick [1987], Froot, and Thaler [1990], and Engel [1996] for surveys of the existing literature.

Estimates for β_{Level} are highly unstable, and depend on the analyzed period, the investment horizon, and the currency pairs. Nevertheless, $\beta_{\text{Level}} = 1$ can often be rejected, while $\beta = 0$ cannot be rejected in many analyses, and the estimate for beta is often negative. For example, the average beta in 75 papers surveyed by Froot and Thaler [1990] is -0.88.

The bulk of the literature analyzed the period from the seventies, using short-term interest rates from one month up to one year, with the main focus on dollar exchange rates. While one might argue such a time horizon was too short for papers published in the eighties, this argument no longer holds, as there exists now nearly 40 years of data since the abandonment of the Bretton-Woods fixed exchange rate system in 1973. However, even recent papers rarely never report positive values for beta.

One of these exceptions is the analysis by Lothian and Wu [2011]. They ran the UIP regression for the dollar-sterling and the franc-sterling exchange rates since 1831, using short-term (maturity below one year) and long-term (maturity above 10 years) interest rates. The beta for the dollar-sterling rate is 0.38 for the long-term rate and 0.14 for the short-term rate, while it is 0.73 and 0.97, respectively, for the franc-sterling rate. In the case of the long-term rate for the franc-sterling, UIP cannot be rejected, while $\beta_{\text{Level}} = 0$ can be rejected. For the short-term rate for the franc-sterling, neither $\beta_{\text{Level}} = 0$ nor $\beta_{\text{Level}} = 1$ can be rejected. In the sub-periods, they only found negative betas in the seventies and eighties, which could explain some of the negative betas in the literature from this area. However, various problems reduce the reliability of the result: interest rates are not equal across the countries, as some include credit risk while others are assumed to be risk-free, and the maturity of the interest rate also differs. Furthermore, there are only two currency pairs, and the data quality of 200-year old data is also questionable, since trading in the pre-computer era was completely different.

Campbell, Koedijk, Lothian, and Mathieu [2007] analyzed 20 currencies against USD from 1976 to 2005, using short-term interest rates. They found slightly positive betas for 6 of 20 currencies but, for 15 of 20 currencies, the hypothesis $\beta_{\text{Level}} = 1$ could be rejected at a 5% significance level. In a second step, they averaged the data over 5, 15, and 30 years. The resulting betas from the cross-panel regressions are positive, at 0.04, 0.69, and 0.58.

Chinn [2006] analyzed the G7 currencies against the USD from 1980-2000, using interest rates of between 3 months and 10 years. While the beta is negative for the panel estimates with interest rates of up to 12 months, it is positive for the 3, 5 and 10-year interest rates, with values of 0.03, 0.67, and 0.68. Chinn concludes than UIP seems to hold better at long horizons than at short ones. He argues that measured interest rates and exchange rates are imperfect measurements of the equilibrium values, but that the errors-in-variable problem is relatively smaller for long-term variables. Nonetheless, he admits that, for long-term rates, the analyzed horizon is rather short, as he holds long-term interest rates always to maturity.

Another positive finding is with long-term interest rates over medium horizons. Alexius and Sellin [2012]) found positive betas by using expected 10-year bond returns over periods from one to 30 weeks. They obtained significant positive betas from the 8- to 30-week horizon for the USD/DEM exchange rate in the sample period from October 1993 to November 1998.

Boudoukh, Richardson, and Whitelaw [2013] ran the UIP regression with one-year forward interest rates, ranging from one to four years forward. Using these forward rates, they obtained positive betas for the exchange rates USD/CHF, USD/DEM, and USD/GBP for the period from 1980-2010. Nonetheless, they noted that the coefficients are noisy, and vary between -0.16 and 3.41 However, this still supports the existing evidence that UIP does not work for short term interest rates, while it seems to be a better fit for long-term interest rates.

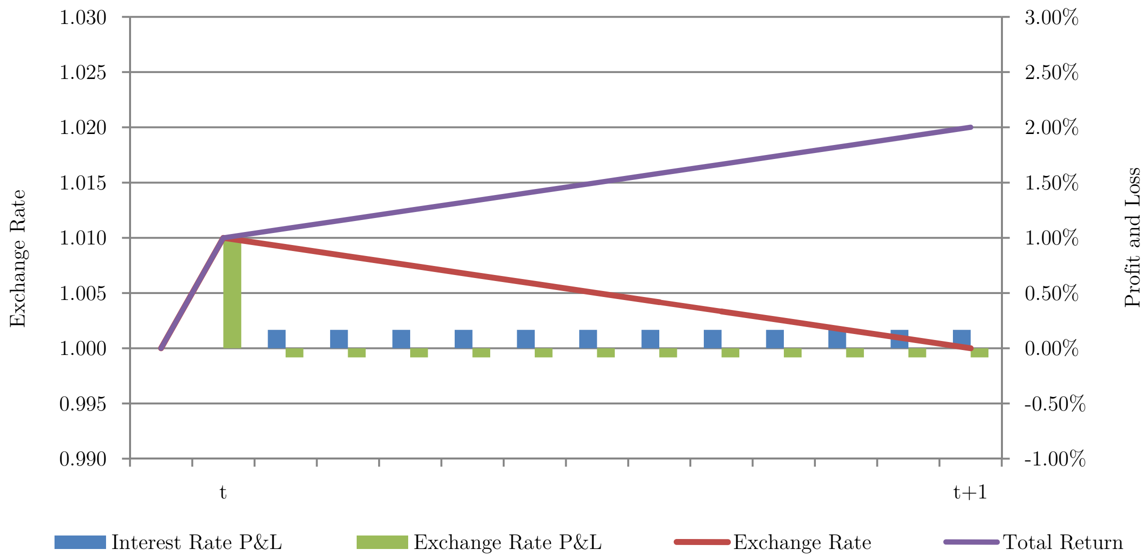
II.2.3 Explanations for UIP failure

In recent years, research has shifted from testing the UIP regression to explaining why UIP fails. There are numerous explanations for such failure, including the risk aversion of investors, peso problems, and distorted beliefs.

Risk aversion
Risk aversion can explain the deviations of beta below 1, but not of those below 0. Two effects of risk aversion could influence the UIP relationship. Firstly, an increase in risk reduces the profitability of the attempt to exploit UIP failure (the carry trade), as the expected return remains the same but, as the risk increases, the return-to-risk ratio decreases. If the return-to-risk ratio is too low, it is not worth exploiting UIP failure when other investments offer a better reward. Secondly, the carry trade is often implemented through leveraged investment, since returns from unleveraged investments are somewhat small. If the risk increases, the levered investors (such as hedge funds) might be forced to reduce their positions, as their brokers require higher margins to cover potential losses. When losses occur, existing positions must be sold to cover the losses, leading to a downward spiral. In this downward spiral, UIP seems to work quite well, as high yielding currencies depreciate, as seen in 2008. However, when the risk decreases again, exchange rates move against UIP, as high yielding currencies also begin to appreciate. Melvin and Taylor [2009] provide a good overview regarding the relationships between currencies, interest rates, and risk.

Given the factor of risk aversion, Figure 1 would change slightly, as presented in Figure 2:

Figure 2: Exchange Rate and Total Return Development with Risk Aversion



This illustration shows the effect of an unexpected interest rate increase on the future exchange rate and interest rate return, with the assumption of risk-averse investors.

In the example of Figure 2, the exchange rate does not initially appreciate by 2%, since return-to-risk ratio is too small after a certain level of the exchange rate. From t to t+1, the total return from the investment in the high-yielding currency generates a profit, since the exchange rate loss is smaller than is the interest rate gain. Nonetheless, this does not explain why most studies observe gains in the exchange rate.

Some people have tested whether risk aversion could explain the failure. If the UIP fails and the beta is below 1, or even below 0, profits can be generated by buying high-yielding currencies and selling low-yielding ones. This strategy is famously known as the carry trade. If the carry trade portfolios generate significantly risk-adjusted profits, then UIP failure cannot be explained via risk alone. Another large body of literature has analyzed this phenomenon and has found evidence against UIP.

Baz, Breedon, Naik, and Peress [2001] report a Sharpe ratio of 0.6, while Ackermann, Pohl, and Schmedders [2012] report a Sharpe ratio[2] of 1.01 before and of 0.93 after transaction cost for a carry-trade portfolio consisting of 11 currencies in the period from 1990-2012. This is compared to a Sharpe ratio of 0.36 for the S&P 500 over the same sample period. If UIP fails, the return from the carry trade should be significantly different from 0, which is the case in most studies. Skewness is positive, while equity markets have negative skewness (S&P 500 was -0.56 over the same period). The high Sharpe ratios suggest that the optimized carry trade is one of the most

[2] The Sharpe ratio is the ratio of excess return to risk.

profitable investment strategies known. It is no accident that Jylhä and Suominen [2011] found that 16% of the overall hedge fund returns are related to the carry trade. The profitability of the carry trade strategy confirms the failure of the puzzle of UIP measured with short-term interest rates.

Clarida, Davis, and Pedersen [2009] went one step further and ran the regression for equation (2) for the 3 long-3 short carry trade portfolio in different volatility regimes. The result is a beta of -3.29 at the low volatility regime, 2.73 at the high volatility regime, and -1.21 on average. They concluded that the beta depends strongly on the volatility regime, which explains part of the failure of UIP in previous studies. However, the average beta is too low to fully explain UIP failure by volatility regimes.

Brunnermeier, Nagel, and Pedersen [2008] analyzed the crash risk of carry trades. They found that high interest rate currencies have negatively skewed returns, while low interest rates currencies have positively skewed ones. Furthermore, they regressed the change of the VIX Index (S&P 500 implied volatility index) to the return of the carry portfolio and found a negative relationship. For both market risk indicators, an increase of risk leads to negative carry returns. For the 3-3 portfolio, the relationship is strongly significant, while this is not the case for the optimized portfolio of Ackermann, Pohl, and Schmedders [2012].

All these studies show that risk has an influence on the UIP regression. Carry returns are sensitive to changes in risk. Investors are risk-averse, but the Sharpe ratios are still too high and the skewness is not sufficiently low to serve as a comprehensive explanation of UIP failure. Thus, it is well worth extending regression (2) by a risk factor.

Peso problems

A peso problem means an event that occurs with a low probability but with a relatively strong influence. A good, recent example is the exchange rate limit between CHF and EUR that was set at 1.20. The Swiss national bank forcefully stated that it would not allow the EUR/CHF exchange rate to drop below 1.20. Since the announcement on September 6 2011, this boundary has held. Nonetheless, the potential remains and, if investors want to protect themselves against a drop below 1.20, they have to pay a premium of 3.21% p.a. for a put option (a European put option with a strike price of 1.20 and a maturity of one year's due date March 26 2012, source ZKB Trading). As long as this does not happen, it is possible to profit from UIP violation with a carry trade, as forward rates have been traded below 1.20, or to simply sell put options with a strike price of 1.20. This strategy would generate steady, small profits, as long as the Swiss national bank defends 1.20. Furthermore, profits would appear to be very low risk ex-post, as the EUR/CHF exchange rate was very stable in 2012 and was close to 1.20. However, in case of the peso event that the Swiss national bank breaks its promise to defend 1.20, the loss would probably be immense.

Burnside, Eichenbaum, Kleshchelsk, and Rebelo [2011] addressed this problem in a more general way. They considered a two-state regime-switching model. In one state, the peso event does not occur and the carry trade is successful. In the other state, the peso event occurs and the carry trade return is negative. They analyzed this empirically by combining the carry trade with option protection, using data from 1987-2009 for 20 different countries. Although the return decreases from 3.0% to 1.6% for the peso-protected portfolio, the Sharpe ratio decreases only slightly, from 0.48 to 0.45, as the volatility also decreases. This means that the cost of protection against worse peso events is less than is the profit that is generated by the carry trade.

Jurek [2008] also analyzed the carry trade with option protection for the period of 1999-2007 for G10 currencies. He concluded that 30-40% of the carry trade profit could be explained by crash risk. Crash-neutral carry portfolios generate significant profits, with Sharpe ratios of between 0.63 and 1.28 for different hedging and portfolio construction strategies, while skewness is positive for most strategies. He stated that the implied volatility should be approximately four times higher than it is for unobserved peso events in order to explain UIP failure and to reduce the profitability of the carry trade sufficiently.

Empirical analysis has not provided sufficient evidence to conclude that UIP failure could be explained by peso events alone. As we have seen, option prices are cheap enough that hedged carry trade strategies still generate profits. Furthermore, we have over 35 years of data including some major crises, such as the LTCM crisis in 1998, the IT Bubble in 2000, the financial crisis in 2008 and the European debt crisis starting in 2010. All these events can be regarded as peso events for a certain currency. While carry returns for affected currencies have been negative during these crises (for example, the JPY appreciated 28% against the AUD in October 2008, which is extremely large compared to the return by the interest differential of roughly 0.5% in that month), returns have subsequently been positive. In addition, given the cross-section of many currencies that have been subject to empirical analysis, it is implausible that all of them would suffer from unobserved peso risks simultaneously.

Distorted Beliefs

Gourinchas and Tornell [2004] offered another explanation for UIP failure. If investors systematically misjudge the duration of the interest difference, the failure of UIP could be explained. By comparing the estimated interest differentials of the investors with those realized, empirical analysis reveals that investors underestimate the persistence of interest differentials, and therefore overestimate transitory interest rate shocks. Since investors realize that interest differentials persist for a period of time, they readjust their future beliefs regarding the persistence, which can lead to an appreciation of the higher-yielding exchange rate.

Let us use the example of Figure 1. Firstly, there is an interest shock of 2% in the foreign interest rate at t. Investors initially believe that the difference will persist for one period. The exchange rate of the foreign currency appreciates by 2% at t, and is expected to depreciate to the original value at t+1. However, at t+1, the foreign interest rate still remains 2% higher than the domestic one. Investors again anticipate that this will only hold for the next period. The effect of the interest rate remaining 2% higher for another period compensates for the depreciation from the initial increase, so that the exchange rate remains unchanged at t+1. The interest difference generates a return of 2% at t+1. Finally, at t+2, investors realize that the interest rate difference will persist for another 4 periods, which would increase the exchange rate by 8%, although this is reduced by 2% by the depreciation caused by the difference in the previous period. Thus, at t+2, the exchange rate increases by 6% and the interest rate gains are again 2%. At t+3, the exchange rate losses are offset by the interest rate gains, as future interest rate expectations do not change. At t+4, the foreign interest rate decreases to the same level as the domestic one. Investors might think that this is only a temporarily effect and that the interest rate will increase again for the last two periods. As a result, the exchange rate falls by only 2% at t+4, but the interest difference is 0. In the last two periods, the interest rate difference remains at 0% and the exchange rate drops by 2% in each period. However, these two periods do not account for UIP regressions, since the interest rate difference is 0. A regression of UIP in this example would result in a beta smaller than 0, since the exchange rate appreciated during the period in which the interest rate difference was positive. Furthermore, an investor who buys the foreign currency at t will make profit.

Figure 3 presents the development of the example described above.

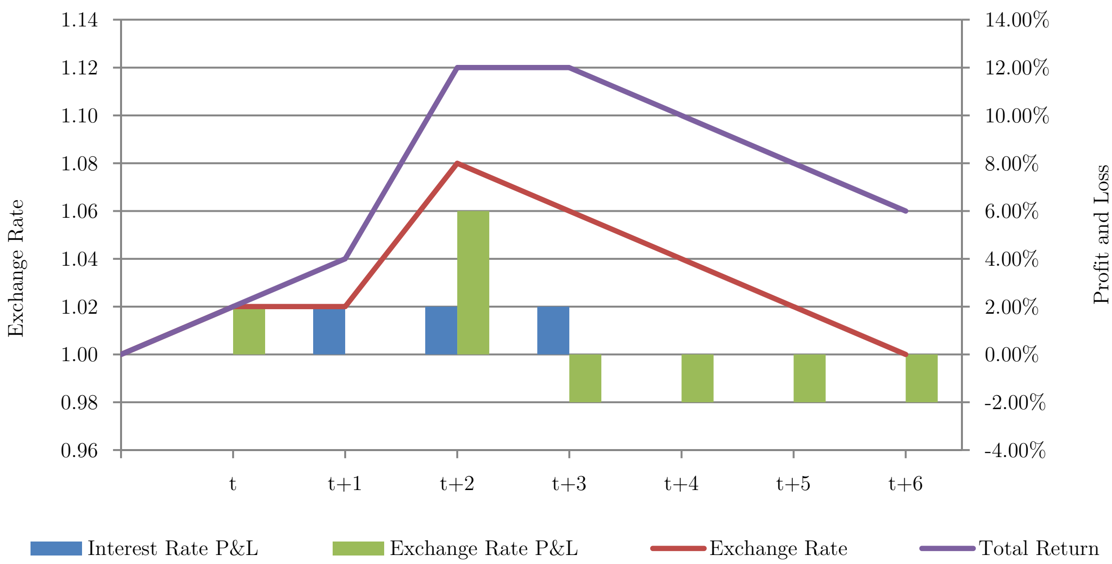
Empirical evidence confirms the hypothesis that investors underestimate the persistence of interest differentials, as Gourinchas and Tornell [2004] showed by comparing interest rate forecasts to realized interest rates.

Davis, Miller, and Prodan [2009] estimated the decay rate of quarterly interest differentials to be between 0.16 and 0.29. They showed that, if the decay rate is correctly anticipated, the beta is 1 and UIP holds. If the decay rate is overestimated, the beta can be negative and UIP is violated.

Craighead, Davis, and Miller [2010] proposed another explanation, in that UIP failure is a mixture of transaction costs and volatility. In the zone of inactivity around the zero interest differential, either the transaction costs outweigh the carry return, or the return is too small compared to the risk. If UIP is regressed over all interest differentials, the inactivity zone invalidates the results. UIP should hold only if the interest differentials are large enough. They also estimate the decay rate of interest differentials which, in their case, is between 3% and 5% for monthly and 11% and 20% for quarterly interest rates. However, the decay rate is very unstable over time, which can explain investors' failure to anticipate the decay rate correctly, and is consistent with unstable betas. Large interest differentials are more likely to decay than are small ones.

While Gourinchas and Tornell [2004] do not explain why investors do not adjust their beliefs, Ilut [2008] does. The explanation is as follows: the investor is not certain regarding the probability that an increase in an interest differential will be temporary or persistent. Under ambiguity aversion, the investor will underinvest in the high interest rate currency (compared to the optimal investment knowing the true probability), as s/he is more pessimistic about the probability that the interest differential is persistent. On average, the investor will be surprised by the longer persistence of the interest rate, and will continue to invest more in the high interest rate currency, which can even lead to a slow appreciation of the high interest rate currency. The theory of distorted beliefs can explain UIP failure and negative betas, if investors cannot correctly estimate the future interest differentials. However, empirical validation of this theory is still scarce.

Figure 3: Exchange Rate and Total Return Development with Distorted Beliefs



This illustration shows the effect of an unexpected interest rate increase on the future exchange rate and the interest rate return, with the assumption of risk neutral investors. However, in this illustration, the duration of the interest rate difference is first underestimated and is then overestimated.

II.2.4 Excursion: Interest Rate Theory

Similar to the uncovered interest rate parity, interest rate theory was also first discussed by Fisher [1896], Shiller [1990] summarized the different theories. A basic theory for the term structure of interest rates is the expectation theory. The simple version of this theory suggests that the long-term interest rates represent the expected future short-term interest rates, assuming risk neutrality. If the long-term interest rates were below the short-term interest rates, investors would expect that the short-term interest rate would fall. Given this theory, the expected profit from an investment in the 10-year bond is the same as a subsequent investment in two 5-year bonds.

The market segmentation theory assumes that the market is segmented into different maturities. These maturities are not substitutes for each other. An example of this theory is that companies prefer to get long-term loans, while retail investors prefer to invest their wealth in short-term bonds or in deposit accounts. This preference increases the long-term interest rates, and decreases the short-term rates.

A more moderate version of the pure expectation and market segmentation theory is the liquidity premium theory. A popular model was developed by Cox, Ingersoll, and Ross [1985]. They state that long-term rates should be equal to the expected short-term rates plus a liquidity premium, as investors prefer short-term bonds. In contrast to the segmentation theory, different maturities are substitutes for each other, although they are not perfect ones. The theory can also account for the risk aversion of investors, as the liquidity premium is also known as the risk premium.

Bansal and Zhou [2002] developed a term structure model with regime shifts between high- and low-volatility regimes. In their two-factor regime-switching model, the risk premium depends on the regime. When confronted with empirical data from the US, they showed that the high-volatility regime coincides with low-risk premiums and economic contraction.

Chinn and Guy [2004] connected the interest rate theory with exchange rate movements. In their model, four different kinds of shocks can occur: exchange market shocks, inflation shocks, output shocks, and term structure shocks. In applying the Taylor Rule, an initial exchange rate depreciation leads to an increase in inflation, output, and interest rates. In the subsequent periods, the exchange rates appreciate back to the initial level, while inflation, output and interest rates fall, which contradicts UIP. The relationship of inflation shocks and output shocks is the reverse, as an initial increase in inflation or output leads to higher interest rates and to an appreciation in the exchange rate. During the subsequent reversion, the inflation, the interest rates, the output, and the exchange rates decrease, which is in line with UIP.

Bekaert, Wei, and Xing [2007] analyzed the failure of the interest rate parity and the term structure simultaneously, using a vector autoregression. They found that UIP is dependent on the currency pair (it holds for the USD-DEM, but fails for the USD-GBP and the DEM-GBP), while the term structure expectation theory fails for all currency pairs. However, they did not see any difference in the time horizon for UIP.

The interest rate theory shows a close relationship between the dynamics of interest rates, exchange rates, and risk premiums. Therefore, an empirical validation of the standard UIP regression (2) is disturbed by many disturbance factors, such as time-varying risk premiums and dynamics in interest rates. To circumvent these disturbance factors, I suggest modifying the uncovered interest rate parity with the dynamics of the interest rate and a risk factor.

II.3 Modified Uncovered Interest Rate Parity

In chapter 2, I pointed out two main reasons for UIP failure in the conventional analysis. Firstly, investors are risk-averse, contrary to UIP assumptions. Secondly, the persistence of interest differentials influences tests of UIP. This theory argues that investors underestimate the persistence, just as they overestimate the decay rate. As a result, the UIP regression should be adjusted to incorporate these effects. Why returns on long-term bonds improve the UIP relationship can be illustrated via perpetual bonds.

II.3.1 Perpetual Bonds

The expectation theory of the interest rate term structure basically states that the returns from every bond should be equal to the risk-free rate. This is illustrated with a perpetuity bond. In an economy with no uncertainty, the price of this perpetuity is:

$$P_t = \sum_{\tau=t}^{\infty} \prod_{\tau'=t}^{\tau} \frac{1}{(1+i_{\tau'})}$$

where 1 is the coupon of the perpetuity paid at time τ. The price change from t to t+1 is

$$\frac{P_{t+1}}{P_t} = (1+i_t)$$

The price of a perpetuity bond in the foreign currency, discounted with the foreign interest rate, is

$$P_t^* = \sum_{\tau=t}^{\infty} \prod_{\tau'=t}^{\tau} \frac{1}{(1+i_{\tau'}^*)}$$

and the price change from t to t+1 is

$$\frac{P_{t+1}^*}{P_t^*} = (1+i_t^*)$$

Thus, if we substitute $(1+\mathrm{i_t})$ with the log prices of $\frac{\mathrm{P_{t+1}}}{\mathrm{P_t}}$ and $(1+\mathrm{i_t^*})$ with the log prices of $\frac{\mathrm{P^*_{t+1}}}{\mathrm{P^*_t}}$ in the UIP equation, we see the following relationship between the domestic and the foreign perpetuity:

$$s_{t+1} - s_t = (p_{t+1} - p_t) - (p^*_{t+1} - p^*_t)$$

This does not only hold for perpetuities. For a bond with maturity T, ∞ has to be replaced with T. Thus, the price of the bond is:

$$B_t = \sum_{\tau=t}^{T} \prod_{\tau'=t}^{\tau} \frac{1}{(1+i_{\tau'})}$$

and the price change is

$$\frac{B_{t+1}}{B_t} = (1+i_t)$$

Therefore, the return of every bond is equal to the risk-free rate under the expectation theory, and that is why bond returns must also fulfill the UIP relationship. We obtain the following relationship between the domestic and the foreign bond:

$$s_{t+1} - s_t = (b_{t+1} - b_t) - (b^*_{t+1} - b^*_t) \tag{7}$$

II.3.2 Modification of UIP

In the standard UIP equation, the maturity of the interest rate is the same as the measurement period for the exchange rate return. Using bond returns with different maturities for return periods shorter than the maturity, the UIP equation is more general. Therefore, I needed to substitute the interest rate by the return of a bond.

$$s_{t+1} - s_t = \alpha + \beta_{Return}(r(t,t+1) - r^*(t,t+1)) + e_{t+1} \tag{8}$$

$(\mathrm{r(t,t+1) - r^*(t,t+1)})$ is the return difference between the return of the domestic and the foreign currency bond. The bond return is defined as follows:

$$r(t,t+1) = C + \frac{PV_{t+1}}{PV_t} - 1 \tag{9}$$

C is the constant coupon rate of the bond, which is paid at t+1, t+2 … T. PV is the present value of the bond,

$$PV_t = \sum_{t=1}^{T} \frac{N*C}{(1+i_t)^t} + \frac{N}{(1+i_T)^T} \tag{10}$$

where N is the notional amount of the bond. As I used an interest rate swaps instead of bonds in this paper, I used the swap rate for C.

The short-term return of the long-term bond can be separated into expected and unexpected returns. The expected return for one month is the 1-month interest rate under the expectation hypothesis of interest rate term structure. The expected return is equal across all maturities if the expectation theory holds perfectly. The expected return refers to the standard UIP regression in regression (2).

If expectation theory does not hold, the unexpected return would contain this information. The unexpected return is the realized bond return minus the expected return. The regression for the unexpected return is

$$s_{t+1} - s_t = \alpha + \beta_{Unexpected}(r(t, t+1) - r^*(t, t+1) - (i_t - i_t^*)) + e_{t+1} \tag{11}$$

If the unexpected return is not zero, the UIP regression cannot hold for all maturities. Short-term interest rates are free from unexpected returns, which could cause their failure in the UIP regression. Therefore, it is reasonable to construct the UIP regression using long-term interest rates over a short horizon, as long-term interest rates capture the effect of unexpected interest returns, while the short horizon generates sufficient data to be of significance.

In the next step, I added a risk factor. I think of risk in terms of the CAPM model. An increase in an asset risk which, in our example is the currency risk, reduces the market return-to-risk ratio respective to the risk premium. Consequently, the price of the asset must decrease until the risk premium is equal to the market risk premium of other assets. Therefore, an increase in risk will reduce the value of high-yielding currencies, leading to a depreciation thereof. Regression (12) includes the changes to the volatility as a risk factor, and is related to the model by Clarida, Davis, and Pedersen [2009] on page 1385, where they model the volatility changes for several regimes. In the regression herein, there are only two regimes, in which the interest rate difference is either positive ($\mathrm{sign}(\mathrm{i_t} - \mathrm{i_t^*}) = 1$) or negative ($\mathrm{sign}(\mathrm{i_t} - \mathrm{i_t^*}) = -1$).

$$s_{t+1} - s_t = \alpha + \beta_{Volatility}\Big((\sigma_{t+1} - \sigma_t) * sign(i_t - i_t^*)\Big) + e_{t+1} \tag{12}$$

The term $\mathrm{sign}(\mathrm{i_t} - \mathrm{i_t^*})$ differentiates whether $(\mathrm{i_t} - \mathrm{i_t^*})$ is positive or negative, since $\beta_{\mathrm{Volatility}}$ is expected to have the opposite sign for negative $(\mathrm{i_t} - \mathrm{i_t^*})$. As σ_t depends on $\mathrm{s_{t+1}} - \mathrm{s_t}$, there is an endogenic problem, and the resulting absolute values have to be interpreted with caution. Nevertheless, the resulting values provide valuable information when we want to determine whether an increase in volatility leads to an appreciation or a depreciation of the currency with the relatively higher interest rate.

With these two factors in place, the standard regression (2) is modified in a multilinear regression, as follows:

$$
\begin{aligned}
&s_{t+1} - s_t = \alpha + \beta_{\mathrm{Return}}(r(t,t+1) - r^*(t,t+1) - i_t^*) + \\
&\beta_{Volatility}\Big((\sigma_{t+1} - \sigma_t) * sign(i_t - i_t^*)\Big) + e_{t+1}
\end{aligned}
\tag{13}
$$

A test of the modified UIP regression is $\alpha = 0$, $\beta_{\mathrm{Return}} > 0$ and $\beta_{\mathrm{Volatility}} > 0$. As investors are risk-averse, I do not require β_{Return} to be equal to 1. However, we can assume that β_{Return} is smaller than 1, because it would be 0 if there were not any trade, as the exchange rate would not move in such a case. As soon as there are trades, β_{Return} will increase towards 1. Lower risk aversions means becoming closer to 1. $\beta_{\mathrm{Volatility}}$ only covers the variation of risk over time, not the risk-aversion itself.

The inclusion of the long-term interest return difference should lead to significant improvement in the UIP regression. Firstly, the magnitude of the long-term interest returns is much higher than it is for the returns of short-term interest differentials. Thus, the disturbance caused by other effects is smaller. Furthermore, distorted beliefs do not influence the regression of long-term interest return differences, because those beliefs are incorporated into the unexpected interest return of long-term interest rates. Secondly, the short-term interest rate is a biased measure of the future interest rate, since investors often anticipate changes to central bank interest rates. Central banks often (but not always) provide hints as to the future development of interest rates. As a result, if a rise in the central bank interest rate is anticipated by all investors, the short-term interest rate will change, while the long-term will not change significantly. Swanson [2006] analyzed the forecast capability of investors in detail. Thirdly, another problem is that interest rates, as well as interest differentials between two currencies, contain unit roots (see, for example, Romero-Ávila [2007] or Meese and Singleton [1982]). However, the return of the long-term interest difference has no unit root.

II.3.3 Data

I used Bloomberg and Datastream as sources of data. The currency universe consists of the US dollar (USD), the Canadian dollar (CAD), the Japanese yen (JPY), the Swiss franc (CHF), the British pound (GBP), the Australian dollar (AUD), and the Euro (EUR)[3].

Spot exchange rates (s_t) against USD are provided by Bloomberg. Cross-exchanges rates are derived from the USD exchange rates. For the (short-term) interest rates (i_t), I used either the official LIBOR rates from Bloomberg with maturities of one month, two months, three months, and six months, or the 12- month interbank rate from Datastream. The advantage of the 12-month interbank rate is the longer horizon. For the long-term interest rate returns $r(t, t+1)$, I used swap rates with maturities of two years, three years, four years, five years, seven years, 10

[3] Before the introduction of the EUR on 1.1.1999, I used the German mark (DEM) instead of the EUR.

years, 15 years, 20 years, and 30 years. I took the 5-year government yield as a comparison, so as to be sure that the swap and bond rates produced similar results. The advantage of the swap rates is that they are actively traded and have consistent terms and conditions, as they are all based on LIBOR rates, and have the same framework dictated by ISDA (International Swaps and Derivatives Association). An important condition is that swaps have a constant maturity. There are trade prices available for the swap rate every day. This is an advantage for government yields, which do not have equal terms and conditions. On the other hand, government yields typically have a longer data horizon.

Perpetual interest rates would be the best proxy for long-term interest returns, but many currencies do not have them. Swap rates with a maturity of 40 and 50 years are also available, although these do not exist for all currencies. The swap rates with maturities above 10 years are available for all currencies, but only for a short time. Furthermore, long-term swap rates are not as liquid as are the 10-year ones. The liquidity of long-term interest rate instruments can be observed via interest rate futures. While there are 10-year bond futures for all G7 currencies, there are only two 30-year bond futures, for USD and for EUR. In 2012, the daily trading volume was 950'000 contracts for the 10-year USD bond future and 330'000 contracts for the USD 30-year future (each traded future is worth roughly $100'000). The daily volume was 660'000 for the EUR 10-year bond future, and 8'000 for the 30-year bond future. Thus, liquidity is much smaller for long-term maturity contracts. Therefore, the 10-year swap rate is considered to be the most reliable proxy for a perpetual bond.

For the volatility, I used the currency volatility index by JP Morgan (JPMVXYG7 Index), which is the average, implied volatility of the G7 currencies. The volatility index began on June 1 1992. Since results depend heavily on the choice of the base currency, I regressed all currency pairs against each other, once in individual regressions for all 21 currency pairs, and once in a fixed-effects cross-panel regression.

Table 1: Descriptive Statistics 1991-2012

	USD	AUD	JPY	GBP	CHF	CAD	EUR
LIBOR 1M	3.52%	5.68%	1.07%	5.08%	2.25%	3.90%	3.71%
Swap 10Y	5.48%	6.94%	2.44%	5.99%	3.68%	5.75%	5.11%
Exchange Rate Return	0.00%	1.36%	2.06%	-0.78%	1.53%	0.70%	0.05%

This table presents the average one month LIBOR interest rate and the average 10-year swap rate from 1991-2012. The exchange rate return is the annual return of the specific currency against the USD.

Table 1 presents the descriptive statistics of the main data in this paper. All currencies, except the GBP, appreciated against the USD. However, the average one-month LIBOR and 10-year swap rates were higher for AUD, GBP, and CAD than they were for USD. The term spread, the difference between the 10-year swap rate and the one-month LIBOR is positive for each currency, and averages 1.46%.

II.3.4 Empirical Analysis of Standard UIP

Most studies test the uncovered interest rate parity with short-term interest rates and regression (2), despite its vulnerability. Table 2 presents the regression results for the standard regression with the dataset of this paper.

Table 2: UIP Standard Regression with 1-Month LIBOR 1987-2012

		R2	α		β_{Level}		Confidence Interval 95%	Observations
Panel		0.00	-		-1.00	(-4.04)	[-1.49 -0.52]	6'552
USD	AUD	0.00	0.00	(0.53)	-0.93	(-1.03)	[-2.71 0.85]	312
USD	JPY	0.01	-0.01	(-2.34)	-1.88	(-2.01)	[-3.72 -0.04]	312
USD	GBP	0.00	-0.00	(-0.12)	0.24	(0.16)	[-2.60 3.08]	312
USD	CHF	0.01	-0.00	(-1.17)	-1.42	(-1.26)	[-3.64 0.80]	312
USD	CAD	0.00	-0.00	(-0.41)	-0.44	(-0.67)	[-1.73 0.85]	312
USD	EUR	0.00	-0.00	(-0.22)	-0.37	(-0.36)	[-2.36 1.63]	312
AUD	JPY	0.00	-0.00	(-0.54)	-0.73	(-0.70)	[-2.81 1.34]	312
AUD	GBP	0.00	0.00	(0.40)	-1.27	(-1.00)	[-3.79 1.24]	312
AUD	CHF	0.00	-0.00	(-0.90)	-1.18	(-1.08)	[-3.32 0.97]	312
AUD	CAD	0.00	-0.00	(-0.06)	-0.48	(-0.50)	[-2.35 1.39]	312
AUD	EUR	0.00	0.00	(0.11)	-0.36	(-0.43)	[-2.05 1.32]	312
JPY	GBP	0.00	0.00	(0.95)	-0.49	(-0.44)	[-2.65 1.67]	312
JPY	CHF	0.01	0.00	(1.63)	-3.32	(-1.91)	[-6.73 0.09]	312
JPY	CAD	0.00	0.01	(1.29)	-1.54	(-1.03)	[-4.48 1.40]	312
JPY	EUR	0.00	0.00	(0.32)	0.28	(0.15)	[-3.40 3.95]	312
GBP	CHF	0.01	-0.01	(-2.18)	-1.60	(-2.14)	[-3.08 -0.13]	312
GBP	CAD	0.01	-0.00	(-1.55)	-2.56	(-1.70)	[-5.52 0.40]	312
GBP	EUR	0.00	-0.00	(-0.65)	-0.52	(-0.74)	[-1.92 0.87]	312
CHF	CAD	0.02	0.01	(2.25)	-3.02	(-2.16)	[-5.77 -0.27]	312
CHF	EUR	0.01	0.00	(2.48)	-2.26	(-1.95)	[-4.54 0.02]	312
CAD	EUR	0.00	-0.00	(-0.06)	-1.14	(-1.01)	[-3.36 1.08]	312

This table shows the results of the standard UIP regression (2) for each currency pair and the panel regression (with fixed effects) of all pairs, using the one month LIBOR level as the explanatory and the one month change of exchange rate as the explained variable during the period from 1987-2012. The reported t-statistics are the Newey-West estimators (Newey and West [1987]), which are adjusted for autocorrelation and heteroskedasticity

I used non-log exchange rates for the entire empirical analysis, as the interest rates and interest rate returns are also discrete. I also computed the results with log data, and the results are very similar.

The results of the panel regression are disappointing. The R-squared is 0.00, and the beta has the wrong sign and is significantly smaller than 0 at the 99% significance level, with a t-value of -4.04. For the individual currency pairs, the resulting betas are far from having a significant beta. The corresponding t-values would be +/-1.97 for the 95% significance level and +/-2.59 for to 99% level. Only USD/GBP and JPY/EUR have a positive beta (0.24 and 0.28, respectively). Three currency pairs (USD/JPY, GBP/CHF, and CHF/CAD) have a beta that is significantly smaller than 0 at the 95% level, while all others are not significantly different from 0 at the 95% significance level. At the 99% significance level, none is significantly different from 0, while four pairs are significantly different from 1. The confidence interval is quite large, which makes a reasonable conclusion difficult. The upper bound of the confidence interval is in the region of 10 out of 21 pairs above 1, which explains why few studies find positive betas for certain time windows. Generally, these results are similar to those in the existing literature, meaning that UIP does not hold at the short horizon with short-term interest rates. In addition, most results even have the wrong sign for UIP to hold.

The results are robust for the use of interest rate maturity and as a source for the one-month frequency. Table 3 presents the panel regression results with different maturities over the horizon from 1991-2012.

Table 3: UIP Panel Regression with Different Interest Rates 1991-2012

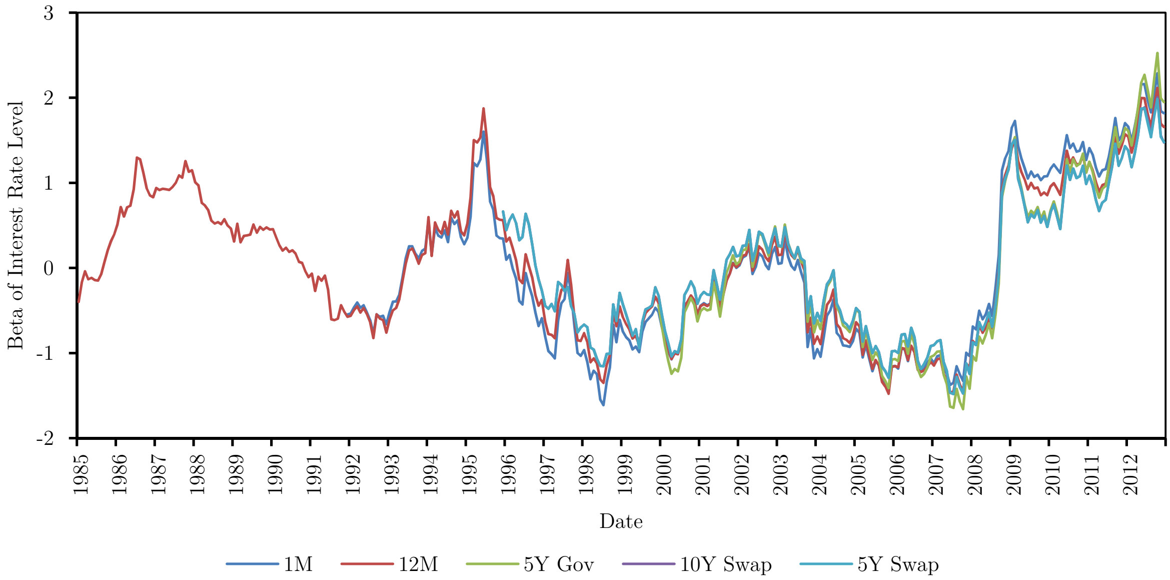
1991-2011	1M	12M	Swap 5y	Swap 10y	Gov 5y*	Swap 5y*
$\boldsymbol{\beta_{Level}}$	-0.95	-1.10	-2.12	-1.72	-1.71	-1.54
t-value	-2.71	-2.96	-3.63	-2.45	-2.12	-2.43

This table shows the results of the panel regression (2) (with fixed effects) for different interest rate maturities, using the level of the interest rate as the explanatory and the one-month change of exchange rate as the explained variable. The period is from 1991-2012, with the exception of Gov 5y* and Swap 5y*, where the period is 1995-2012. The reported t-statistics are Newey West estimators, which are adjusted for autocorrelation and heteroskedasticity problems.

The beta is negative for all maturities, and is even more negative for long-term maturities at the short horizon. The difference between the regression with 5-year government rates to 5-year swap rates is small. However, it should be pointed out that only the level of the long-term interest rate is regressed against the exchange rate change over one month in this instance.

While most previous studies reported negative betas or values around 0, the occasional study also found positive betas. One reason for this is that the results depend on the choice of currencies pairs, as we have seen in Table 2. Table 3 shows that the choice of interest rate maturity also matters to some extent, while swap or government rates produce similar results. Another issue is that the relationship is very unstable over time, as presented in Figure 4. Figure 4 presents the 5-year rolling regression of the particular rates. A 5-year rolling regression is regression (2) for 60 months, which means that the first point for the 12-month regression is the beta over the period from December 31 1979 to December 31 1984. The second point in the graph is the regression results from January 31 1980- January 31 1985. Not all regressions start in December 31 1984, as not all interest rates are available from the beginning of the year.

Figure 4: 5-Year Rolling UIP Panel Regression with Different Interest Rates



This figure shows the 5-year rolling panel standard UIP regression (2) (with fixed effects) for different interest rate maturities, using the level of the interest rate as the explanatory and the one-month change of exchange rate as the explained variable. The period is from 1980-2012. The reported t-statistics are Newey West estimators, which are adjusted for autocorrelation and heteroskedasticity problems.

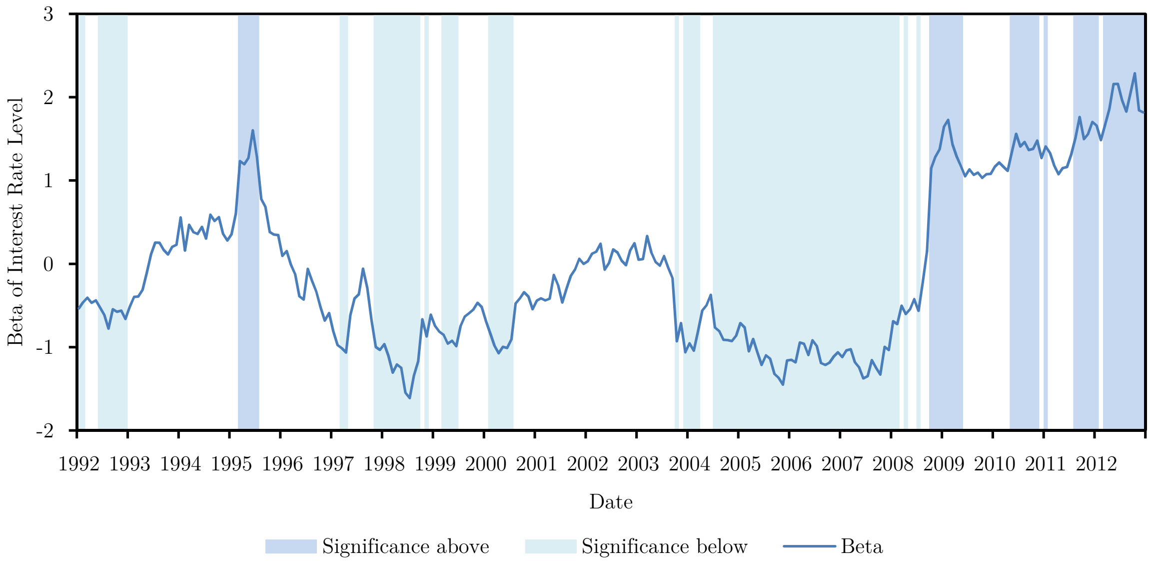
Figure 5 presents the results of the rolling regression for the one- month interest rates, and shows the periods in which the beta is significant at a 95% significance level.

In the period during 1994-1995, the beta was significantly positive. However, from 1996-2007 it was often significantly negative, contrary to UIP. Between 2007 and 2008, the beta increased sharply to a level above 1. This was mainly driven by the financial crisis in 2008, when a massive unwinding of carry trades led to the crash of high-yielding currencies, and to an appreciation of low-yielding currencies (see Melvin and Taylor [2009] for details).

Figure 6 presents the results of the 2-year rolling regression for the one-month interest rates, and shows the periods where beta is significant at a 95% significance level.

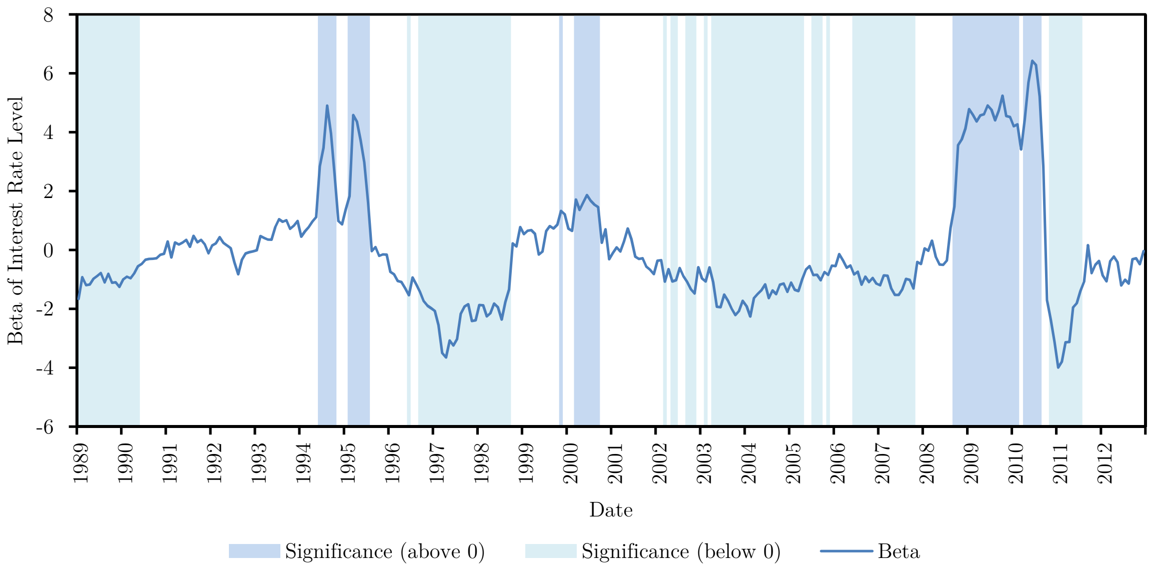
The 2-year regression has the advantage of providing more detail for the year that influences the beta, but has the disadvantage of having fewer data points in order to estimate the regression. During the financial crisis of 2008 and 2009, the beta was extremely positive (up to +6), but became negative again in 2009 and 2010, which means the positive beta in 2008/2009 was only a temporary phenomenon. The rapid switch from positive significance to negative significance within two months is impressive.

Figure 5: 5-Year Rolling UIP Panel Regression with 1-Month Interest



This figure shows 5-year rolling panel regression (2) (no fixed effects as the horizon is too short) for the one-month interest rate. The period is from 1987-2012. The reported t-statistics are Newey-West estimators, which are adjusted for autocorrelation and heteroskedasticity problems.

Figure 6: 2-Year Rolling UIP Panel Regression with 1 Month Interest Rate



This figure shows the beta for the 2-year rolling panel regression (no fixed-effects as the horizon is too short) of the 1-month interest rates, and shows the periods where the beta is significant at a 95% significance level. A 2-year rolling regression is regression (2) for 24 months: the first observation for the monthly regression is the beta over the period from January 1987 to December 1988, and the last point is the beta over the period from January 2011 to December 2012. The significance level is measured using Newey-West estimators.

Table 4 presents the results of the regression of the 5-year interest rates against the exchange rate return over five years, from 1980-2012. This is the same regression as in Chinn [2006], who found positive betas. For direct comparability, I used the same maturity, the 5-year swap rate (Chinn used the 5-year government bond rate). In the event that the 5-year swap rate was not available, I used the 5-year government rate or the 12-month interbank rate. I repeated the regression with the 12-month interbank rate and obtained similar results (the beta of the panel regression is 0.25 with a t-value of 2.92 for the 12-month rate).

Table 4: UIP Long-Term Regression with 5-Year Interest Rates 1980-2012

		R2	α		β_{Level}		Confidence Interval 95%	Observations
Panel		0.02	-		0.44	(3.56)	[0.20 0.68]	2'373
USD	AUD	0.02	1.00	(0.11)	-0.52	(-0.92)	[-1.63 0.59]	113
USD	JPY	0.01	-0.99	(-0.09)	0.25	(0.49)	[-0.75 1.25]	113
USD	GBP	0.11	-0.44	(-0.02)	1.24	(1.55)	[-0.34 2.82]	113
USD	CHF	0.01	-0.84	(-0.05)	0.29	(0.80)	[-0.43 1.01]	113
USD	CAD	0.23	-2.04	(-0.06)	1.43	(3.79)	[0.68 2.18]	113
USD	EUR	0.11	0.37	(0.02)	0.96	(2.62)	[0.23 1.69]	113
AUD	JPY	0.02	0.34	(0.05)	0.68	(1.26)	[-0.39 1.74]	113
AUD	GBP	0.04	1.21	(0.06)	0.63	(2.30)	[0.09 1.17]	113
AUD	CHF	0.29	2.00	(0.13)	0.98	(3.33)	[0.40 1.57]	113
AUD	CAD	0.10	-2.03	(-0.07)	-0.61	(-2.07)	[-1.20 -0.03]	113
AUD	EUR	0.32	2.09	(0.11)	1.05	(3.79)	[0.50 1.60]	113
JPY	GBP	0.00	1.14	(0.36)	-0.44	(-0.32)	[-3.16 2.29]	113
JPY	CHF	0.24	3.58	(0.09)	-1.06	(-3.64)	[-1.64 -0.48]	113
JPY	CAD	0.02	0.27	(0.05)	0.73	(0.92)	[-0.84 2.31]	113
JPY	EUR	0.04	2.86	(0.17)	-0.54	(-1.02)	[-1.59 0.51]	113
GBP	CHF	0.02	-0.72	(-0.06)	0.30	(1.07)	[-0.25 0.85]	113
GBP	CAD	0.06	-0.64	(-0.02)	0.76	(1.49)	[-0.25 1.78]	113
GBP	EUR	0.07	-0.09	(-0.01)	0.63	(2.17)	[0.05 1.20]	113
CHF	CAD	0.08	0.24	(0.02)	0.58	(1.85)	[-0.04 1.21]	113
CHF	EUR	0.02	0.44	(0.01)	0.41	(1.50)	[-0.13 0.94]	113
CAD	EUR	0.20	1.83	(0.07)	0.99	(3.72)	[0.46 1.52]	113

This table shows the results of regression (2) for each currency pair and the panel regression (fixed-effects) of all pairs, using the 5-year interest level (multiplied by 5) as the explanatory and the 5-year change of exchange rate as the explained variable for the period from 1980-2012. The data are measured at a quarterly frequency, which means only 6 of 113 observations are non-overlapping periods. The reported t- statistics are Newey-West estimators.

The long-term UIP regression has a positive beta in the panel regression. This confirms the results of Chinn, who also reported positive betas. Four currency pairs are significantly positive at the 99% significance level, and another three are significant at the 95% significance level. Only one currency pair (JPY/CHF) has a significantly negative beta at 99%, and the

AUD/CAD is significantly negative at 95%. Another three pairs have negative betas. Six currency pairs are significantly different from 1.

However, the positive beta in the panel regression holds only over the entire sample horizon. For example, in the sample between 1990 and 2012, the resulting beta is -0.05 (-0.17). Since the end of Chinn's data horizon, from 2001-2012, the beta of the panel regression is even significantly negative, with a beta of -1.49 and a t value of -3.24. Thus, the long-term UIP with 5-year interest rates did not work during the past two decades. Furthermore, the data horizon is quite short for 5-year regressions, as there are only 33 years of historical data, which results in slightly more than 6 of 113 non-overlapping periods.

Nonetheless, there is still evidence that the UIP relationship works better with long-term interest rates and over longer horizons. Chinn and Meredith [2004] explained the difference between short- and long-term regression results via the strong influence of the central bank on the short-term interest rates, while long-term interest rates can only be influenced indirectly (apart from the quantitative easing programs of FED, ECB and the Bank of England, which started buying long-term government bonds in the aftermath of the crisis in 2008). Thus, long-term interest rates should be less endogenous than should short-term ones.

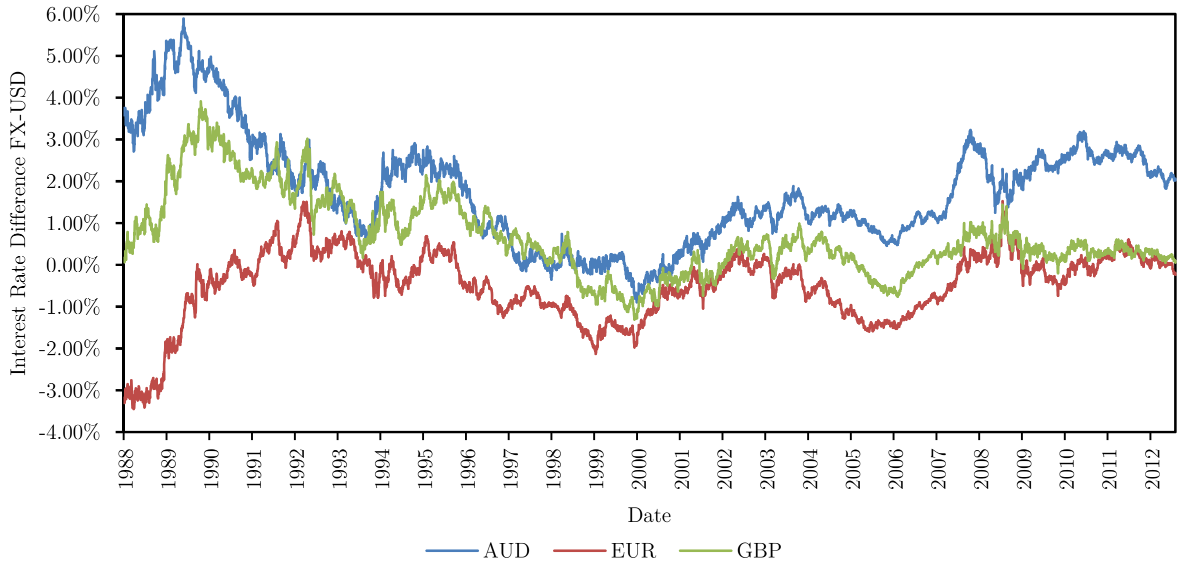
II.3.5 Empirical Analysis of Modified UIP

An explanation for the different results over different periods is that the interest differentials are not constant. Figure 7 presents the development of the 10-year interest rate differences of the AUD, the EUR, and the GBP against the USD. The interest rate differences are also unstable, and thus distort the estimates of the betas.

Figure 8 presents the mean and standard deviation adjusted development of the AUD-USD exchange rate and the interest rate difference between AUD and USD.

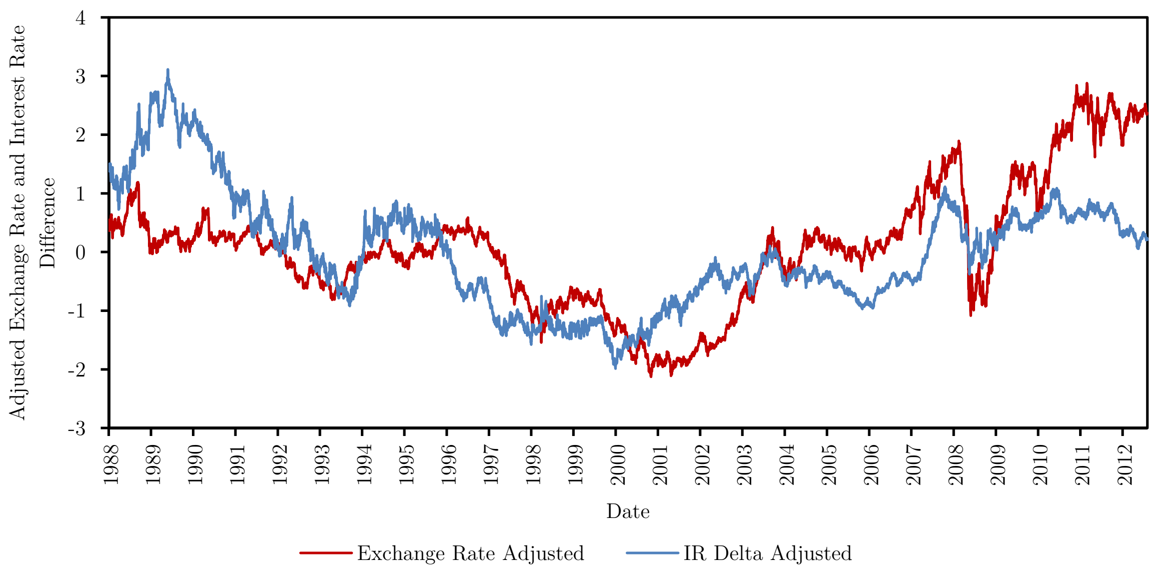
The (inverted) exchange rate and the interest rate difference follow a similar pattern, and seem to be correlated. When the interest rate difference decreased from 1988 to 2000, the exchange rate also depreciated, as predicted by UIP. At the same time, when the interest rate difference increased from 2000 to 2007, the exchange rate also appreciated. Thus, although the interest rate difference is positive from 2001 to 2007 and the usual UIP would suggest that the exchange rate should depreciate, the exchange rate actually appreciated, since the effect of the increasing interest rate difference dominated the depreciation effect from the level of the exchange rate. As the change in the interest rate difference also influences the exchange rate, it should therefore be incorporated into the regression, as suggested in formulas (8) and (13).

Figure 7: 10-Year Interest Rates Differences



This figure shows the development of the interest rate differentials of the AUD, the EUR, and the GBP against the USD. The period is from 1988-2012.

Figure 8: Adjusted 10-Year Interest Rate Differences and Exchange Rate of AUD-USD



This figure shows the mean and standard deviation adjusted AUD-USD exchange rate and its interest rate differential. The period is from 1988-2012.

Table 5 presents the results of regression (8) using the return difference between two currencies' 10-year swap rate over the period 1991-2012.

Table 5: 10-Year Swap Return Difference Regression 1991-2012

		R2	α		β_{Return}		Confidence Interval 95%	Observations
Panel		0.03	-		0.34	(8.94)	[0.27 0.42]	5'544
USD	AUD	0.02	-0.00	(-0.51)	0.33	(1.66)	[-0.06 0.72]	264
USD	JPY	0.04	-0.00	(-0.28)	0.28	(2.77)	[0.08 0.48]	264
USD	GBP	0.10	0.00	(0.25)	0.49	(3.46)	[0.21 0.77]	264
USD	CHF	0.08	-0.00	(-0.02)	0.52	(4.21)	[0.27 0.76]	264
USD	CAD	0.00	-0.00	(-0.18)	-0.10	(-0.77)	[-0.36 0.16]	264
USD	EUR	0.12	0.00	(0.23)	0.67	(5.84)	[0.44 0.89]	264
AUD	JPY	0.03	0.00	(0.66)	0.37	(2.03)	[0.01 0.72]	264
AUD	GBP	0.03	0.00	(1.37)	0.30	(1.96)	[-0.00 0.59]	264
AUD	CHF	0.04	0.00	(0.67)	0.34	(2.46)	[0.07 0.61]	264
AUD	CAD	0.02	0.00	(0.77)	0.26	(1.53)	[-0.07 0.58]	264
AUD	EUR	0.02	0.00	(1.02)	0.27	(1.95)	[-0.00 0.55]	264
JPY	GBP	0.05	0.00	(0.55)	0.43	(3.20)	[0.17 0.69]	264
JPY	CHF	0.06	0.00	(0.21)	0.48	(3.70)	[0.22 0.73]	264
JPY	CAD	0.01	0.00	(0.51)	0.17	(1.11)	[-0.13 0.48]	264
JPY	EUR	0.05	0.00	(0.47)	0.47	(3.51)	[0.20 0.73]	264
GBP	CHF	0.03	-0.00	(-0.45)	0.34	(1.48)	[-0.11 0.78]	264
GBP	CAD	0.03	-0.00	(-0.46)	0.32	(2.43)	[0.06 0.58]	264
GBP	EUR	0.04	0.00	(0.03)	0.42	(1.90)	[-0.02 0.86]	264
CHF	CAD	0.02	0.00	(0.31)	0.29	(1.75)	[-0.04 0.62]	264
CHF	EUR	0.00	0.00	(1.54)	0.09	(0.64)	[-0.18 0.35]	264
CAD	EUR	0.01	0.00	(0.62)	0.24	(1.45)	[-0.09 0.56]	264

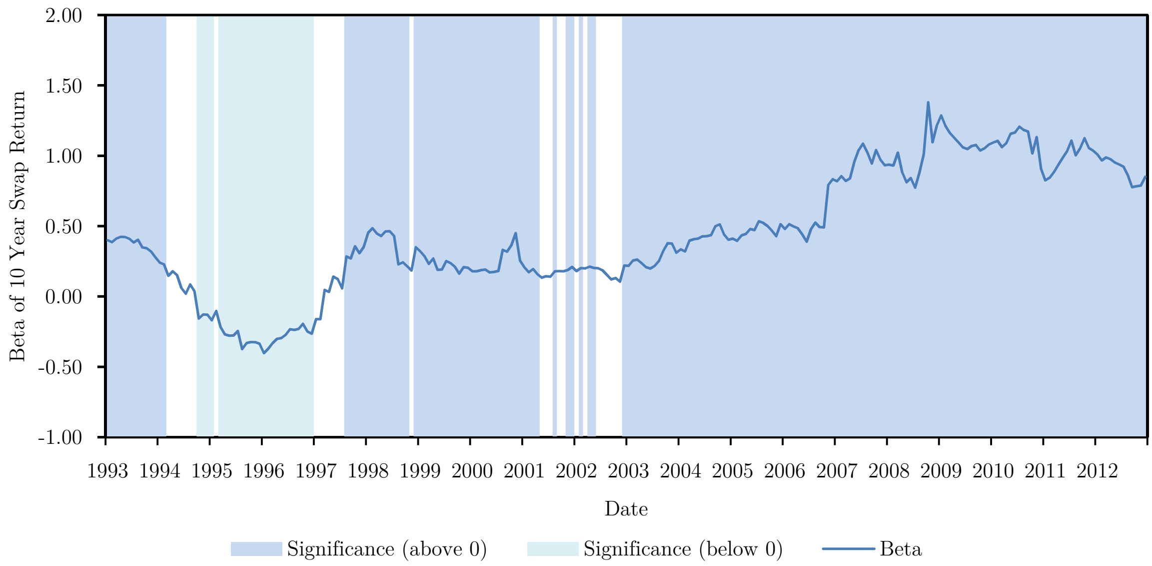
This table shows the results of regression (8) for each currency pair and the panel regression (fixed-effects) of all pairs, using the monthly 10-year swap return as the explanatory and the 1-month change of the exchange rate as the explained variable in the period from 1991-2012. The reported t-statistics are Newey-West estimators.

The results of regression (8) are favorable for the UIP theory. The R-squared for the panel regression is at least 0.03, although this figure is still low. The beta is positive, as expected, which means that if the swap return difference between two countries is positive, then the exchange rate between those two countries will depreciate. β_{Return} is significantly positive at the 99% level (t-value is 8.94). The theoretical risk-neutral value of the beta would be 1, as a decrease in the interest rate by 1% should depreciate the currency by the present value of this 10-year change. However, with the risk aversion and other distortions, it is not surprising that the beta is well below 1.

In the individual regression, only one currency pair (USD/CAD) has a negative beta, which is most likely caused by the close economic relationships of those two countries. Seven currency pairs have positive betas at a 99% significance level and another three at 95%, leaving half of the pairs without a significant beta. These results are also robust for the choice of the interest rate maturity (12 months, 5 years), although the 12-month interest rate clearly has a lower beta and significance level than do the others.

The relationship between the swap return and the exchange rate became stronger over the sample. Figure 9 presents the 2-year rolling regression of the 10-year swap return differences, and shows the periods in which the beta is significant at a 95% significance level.

Figure 9: 2-Year Rolling Regression 10 Year Swap Return Difference



This figure shows the beta of the 2-year rolling panel regression (no fixed-effects as the horizon is too short), using the monthly 10-year swap return as the explanatory and the 1-month change of the exchange rate as the explained variable for the period from 1991-2012. The shaded areas are the periods in which the beta is significantly different from zero, at a 95% significance level. A 2-year rolling regression is regression (8) for 24 months, which means the first point for the 1-month regression is the beta over the period from January 1991 to December 1992, and the last point is the beta over the period from January 2011 to December 2012. The significance level is measured with Newey-West estimators.

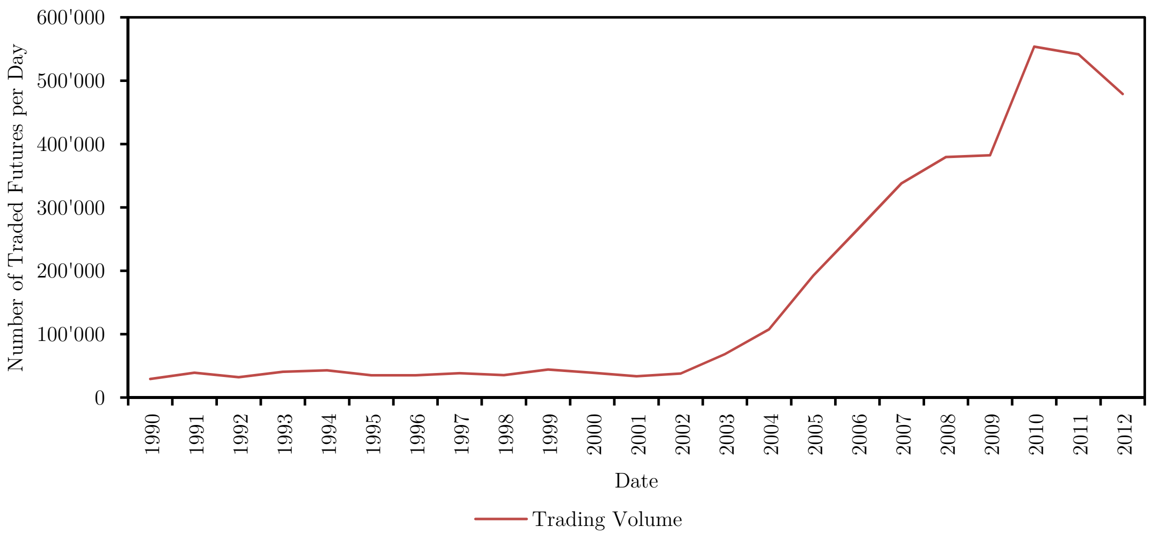
The 2-year regression began to be significantly positive in 1997, which is the regression period from 1995-1997. Before that, it was significantly positive from 1992-1994 and significantly negative from 1994-1996. However, since 1997, the beta remained significantly positive, although at a moderate level between 0 and 0.5, until 2006. Thus, the beta already became much more positive between 0.5 and 0.75, in 2006 (period 2004-2006). Since then, it has settled at levels between 0.75 and 1.25. An important observation is that it did not rise further after the financial crisis of 2009 and 2010, contrary to the beta of the UIP regression in Figure 6.

What happened during the crisis? Before the crisis, interest differentials had been high, and currencies with high interest rates had been at elevated values. During the crisis, high interest rate currencies depreciated dramatically, and so did their interest rates (thus, the long-term interest return difference was positive). Therefore, during this phase, exchange rates behaved as predicted by UIP, in that high interest rate currencies should depreciate. However, after the financial crisis in 2008, interest rates differentials rose again (thus, the long-term interest return difference was negative), and so did the exchange rates. While UIP in the standard form would have suggested that the AUD-USD exchange rate should have depreciated further, since the interest differential was always positive, the exchange rate appreciated strongly. Figure 9 shows that the rise can be explained via the rising interest differential, while the effect of UIP in the standard form is unclear (Figure 5 and Figure 6).

Why the beta decreased dramatically, beginning at the end of 2004-2006 period, is an open question. The sharp increase in the panel regression was not caused by a single currency, but occurred for most currencies, and the effect was more pronounced for currencies with high interest differentials. A possible explanation could be that the market has become more efficient as a greater number of active investors invested in currencies. A possible indication of an increase in the number of active investors is the sharp increase in trading volumes, and open interest in the currency futures market. Although currency futures volumes are only a small fraction of the total currency trading volume, they are typically used by speculative investors. Furthermore, only futures data are available on an annual basis.

The trading volume of the total currency market is reported every three years by the Triennial Survey, and shows a similar development. Figure 10 presents the trading volume of currency futures. The trading volume was stable, between 30’000 and 50’000 traded contracts per day (with an approximate worth of $4 billion), until 2002. However, between 2002 and 2006, the trading volume increased more than tenfold, to 500’000 contracts per day.

Figure 10: Daily Trading Volume in Currency Futures

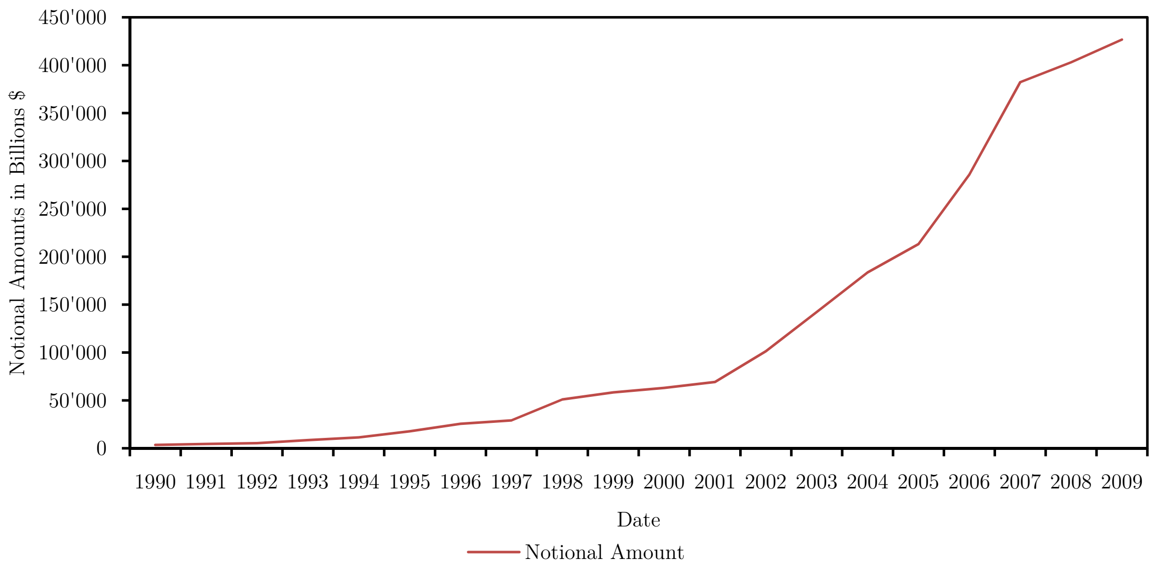


This figure shows the daily trading volume of the six biggest currency futures from 1990 to 2012 (all against the USD, and each future has a value of 100'000 times the exchange rate).

This is consistent with the analysis by King and Rime [2010], who showed that the share of financial institutions in the trading volume of the entire currency market increased dramatically between 2001 and 2010. They argued that the reason for the increase in trading was that financial institutions have had better electronic access during the past 10 years. Until 1999, electronic trading was only available for interbank trading (starting in 1992 with Reuters 2000); thus, customers had to pay relatively high bid-ask spreads in the nontransparent market. In 1999, Currenex started as the first multibank platform available for customers. Other platforms followed, including FXConnect in 2000, BARX in 2001, Autobahn in 2002, and Velocity in 2006.

Meanwhile, the volume of interest rate swaps also increased significantly. In 1990, the year-end amount of the outstanding notional amount was \$ 3400 billion[4], and this amount increased to \$ 426’700 billion in 2009. Data that are more recent are not available. The notional amount includes interest rate swaps, interest rate options, and cross-currency swaps. The share of the interest rate swaps was 77% in 1997, and no data that are more recent are available.

Figure 11: Notional Amount of Outstanding Interest Rate Swaps



This figure shows the year-end notional amount of interest rate swaps, interest rate options, and cross-currency swaps since 1990.

II.3.6 UIP – Change of Volatility Regression

Another problem that distorts the results of the UIP regression is the risk aversion of the investors. As we have seen, a possible way of measuring this effect is regression (12). Table 6

[4] Source: http://www.isda.org/statistics/historical.html

presents the results of this regression, with the G7 implied currency volatility index by JP Morgan as a measure of the volatility, and the 1-month interest differential as a measure of whether the interest differential is positive or negative.

Table 6: Change of Implied Volatility Regression 1993-2012

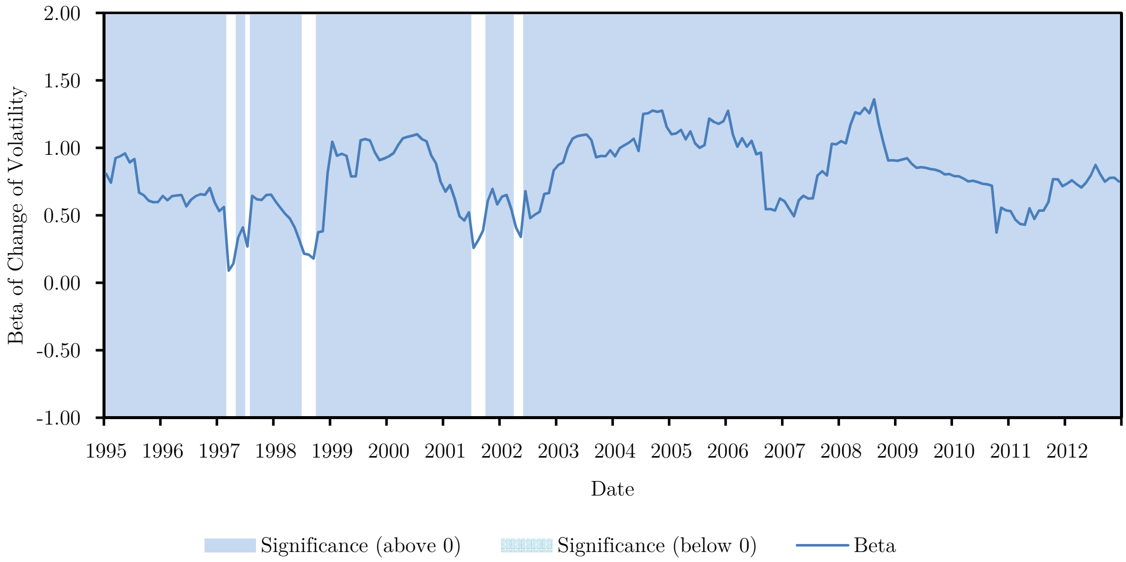
		R2	α		$\beta_{Volatility}$		Confidence Interval 95%	Observations
Panel		0.08	-		0.78	10.27	[0.63 0.93]	5040
USD	AUD	0.15	-0.00	(-0.43)	1.22	(4.13)	[0.64 1.80]	240
USD	JPY	0.11	-0.00	(-0.58)	0.91	(5.39)	[0.58 1.25]	240
USD	GBP	0.07	-0.00	(-0.02)	0.56	(2.94)	[0.18 0.93]	240
USD	CHF	0.00	-0.00	(-0.75)	0.16	(0.59)	[-0.38 0.71]	240
USD	CAD	0.01	-0.00	(-0.58)	-0.24	(-0.69)	[-0.91 0.44]	240
USD	EUR	0.03	-0.00	(-0.11)	0.42	(1.38)	[-0.18 1.02]	240
AUD	JPY	0.31	0.00	(0.38)	2.18	(11.37)	[1.80 2.56]	240
AUD	GBP	0.07	0.00	(0.86)	0.74	(5.81)	[0.49 0.99]	240
AUD	CHF	0.23	0.00	(0.08)	1.50	(9.66)	[1.19 1.81]	240
AUD	CAD	0.07	0.00	(0.62)	0.59	(5.17)	[0.37 0.82]	240
AUD	EUR	0.15	0.00	(1.00)	1.05	(5.78)	[0.70 1.41]	240
JPY	GBP	0.17	0.00	(0.92)	1.40	(7.44)	[1.03 1.78]	240
JPY	CHF	0.06	0.00	(0.15)	0.79	(3.83)	[0.38 1.19]	240
JPY	CAD	0.23	0.00	(0.69)	1.72	(8.03)	[1.30 2.14]	240
JPY	EUR	0.15	0.00	(0.90)	1.28	(4.88)	[0.77 1.80]	240
GBP	CHF	0.06	-0.00	(-0.84)	0.57	(4.04)	[0.29 0.85]	240
GBP	CAD	0.01	-0.00	(-0.32)	-0.20	(-1.58)	[-0.46 0.05]	240
GBP	EUR	0.00	0.00	(0.15)	0.08	(0.64)	[-0.17 0.34]	240
CHF	CAD	0.09	0.00	(0.84)	0.90	(4.38)	[0.49 1.30]	240
CHF	EUR	0.12	0.00	(2.12)	0.48	(3.51)	[0.21 0.74]	240
CAD	EUR	0.01	0.00	(0.59)	0.22	(1.13)	[-0.16 0.61]	240

This table shows the results of regression (12) for each currency pair, and the panel regression (fixed-effects) of all pairs, using the 1-month change of the implied volatility (multiplied by the sign of the interest differential) as the explanatory and the 1-month change of the exchange rate as the explained variable in the period from 1993-2012. The reported t-statistics are Newey-West estimators.

The results in Table 7 confirm the hypothesis that volatility has a strong impact on the UIP relationship. When the interest differential is positive and volatility increases, the high interest rate currency depreciates, while it appreciates when the volatility decreases. The beta from the panel regression is 0.78, and has a significantly high t-value of 10.27, while the R-squared is 0.08. With regard to the individual currency pairs, only the GBP/CAD and the USD/CAD are negative, but insignificantly so. Another four pairs (USD/GBP, USD/CHF, USD/EUR, GBP/EUR, and CAD/EUR) have positive, insignificant betas, while all others (15 of 21) are significant at the 99% level.

Figure 12 presents the results of the 2-year rolling regression of the change of implied volatility.

Figure 12: Rolling Beta Change of Implied Volatility Panel Regression



This figure shows the beta for the 2-year rolling panel regression (no fixed-effects, as the horizon is too short), regressing the 1-month change of implied volatility (multiplied by the sign of the interest differential) on the 1-month change of the exchange rate over the period from 1993-2012. The shaded areas are the periods in which the beta is significant at a 95% significance level. A 2-year rolling regression is regression (12) for 24 months, which means the first point for the 1-month regression is the beta over the period from January 1991 to December 1992, and the last point is the beta over the period from January 2011 to December 2012. The significance level is measured with Newey-West estimators.

The rolling regression reveals that the beta of the change of implied volatility was always positive, with values between 0.1 and 1.4. Thus, volatility has always had a strong impact on the exchange rate, as expected from the theory of risk aversion. This also confirms the UIP theory to some extent, in the sense that there seem to be investors who are trying to exploit the UIP failure, pushing exchange rates closer to UIP and abandoning their trades when the risk increases.

II.3.7 UIP – Multiple Regression

As we have seen in chapters II.3.4 to II.3.6, the long-term interest return difference and the change of volatility are generally both very significant, while the level of the interest differential is not significant. Thus, it is well worth looking at a multiple regression. However, both β_{Level} and β_{Return} measure the interest rate return, with the distinction that β_{Level} measures the return of a short-term interest rate, and β_{Return} of a long-term interest rate. As β_{Level} and β_{Return} are not independent variables, I will only include β_{Return} and $\beta_{\text{Volatility}}$ in the multilinear regression.

Table 7 presents the results of the multilinear regression (13). The results are similar to those of the single regressions. The R-squared is relatively high, 0.10 in the panel regression and 0.34 for the highest individual currency pair AUD-JPY, in contrast to the standard UIP regression that achieves values of 0.00 or 0.01. The 10-year swap return difference and the change of volatility are both highly significant. The Durbin-Watson statistic gives a value of 2.05, which means there is no autocorrelation.

Table 7: Multilinear Regression 1993-2012

		R2	α		β_{Return}		$\beta_{Volatility}$		Observations
Panel		0.10	-		0.30	(7.81)	0.75	(10.04)	5'040
USD	AUD	0.17	-0.00	(-0.60)	0.28	(1.58)	1.16	(4.23)	240
USD	JPY	0.14	-0.00	(-0.24)	0.27	(2.81)	0.90	(5.25)	240
USD	GBP	0.11	-0.00	(-0.19)	0.32	(3.38)	0.53	(2.99)	240
USD	CHF	0.05	-0.00	(-0.45)	0.41	(2.97)	0.11	(0.37)	240
USD	CAD	0.01	-0.00	(-0.57)	-0.05	(-0.36)	-0.24	(-0.71)	240
USD	EUR	0.11	-0.00	(-0.08)	0.55	(4.92)	0.39	(1.38)	240
AUD	JPY	0.34	0.00	(0.93)	0.30	(1.99)	2.12	(11.40)	240
AUD	GBP	0.07	0.00	(0.90)	0.11	(0.74)	0.71	(5.94)	240
AUD	CHF	0.24	0.00	(0.35)	0.20	(1.51)	1.44	(8.28)	240
AUD	CAD	0.10	0.00	(0.85)	0.36	(2.15)	0.54	(5.05)	240
AUD	EUR	0.16	0.00	(1.13)	0.18	(1.31)	1.02	(5.36)	240
JPY	GBP	0.20	0.00	(0.40)	0.34	(2.64)	1.38	(6.94)	240
JPY	CHF	0.11	-0.00	(-0.16)	0.46	(2.84)	0.83	(3.89)	240
JPY	CAD	0.24	0.00	(0.43)	0.18	(1.25)	1.70	(7.67)	240
JPY	EUR	0.18	0.00	(0.43)	0.39	(3.38)	1.27	(4.86)	240
GBP	CHF	0.09	-0.00	(-0.37)	0.36	(1.45)	0.51	(3.43)	240
GBP	CAD	0.02	-0.00	(-0.29)	0.19	(1.47)	-0.18	(-1.35)	240
GBP	EUR	0.06	0.00	(0.46)	0.55	(2.07)	0.06	(0.49)	240
CHF	CAD	0.10	0.00	(0.61)	0.21	(1.15)	0.86	(3.96)	240
CHF	EUR	0.12	0.00	(2.06)	0.07	(0.62)	0.47	(3.71)	240
CAD	EUR	0.02	0.00	(0.64)	0.26	(1.43)	0.23	(1.20)	240

This table shows the results of multilinear regression (13) for each currency pair and the panel regression (fixed-effects) of all pairs, using the monthly 10-year swap return as the interest return (β_{Return}), and the 1-month change of the implied volatility ($\beta_{Volatility}$) as the explanatory variables, and the 1-month change of the exchange rate as the explained variable. The period is from 1993-2012. The reported t-statistics are Newey-West estimators.

II.3.8 Sensitivity Analysis

If the average interest rate difference is small, the results can be distorted, because UIP is not the dominant factor in this area. Returns from carry trades are very small if the interest differentials are small, or might even be negative if the transaction costs are incorporated. Thus, it is not worth exploiting the profit opportunities. Table 8 presents the results of regression (13), considering only currency pairs where the average interest differential over the sample period is smaller than 1%, is at least 1%, and is at least 2%, respectively.

Table 8: Multilinear Regression Constrained to Average Interest Differential 1993-2012

		R2	β_{Return}		$\beta_{Volatility}$		Observations
Panel	All	0.10	0.30	(7.81)	0.75	(10.04)	5‘040
Panel	>2%	0.19	0.28	(5.14)	1.27	(13.02)	1‘920
Panel	>1%	0.15	0.29	(6.61)	1.02	(12.63)	3‘360
Panel	<=1%	0.03	0.28	(4.56)	0.22	(2.09)	1‘680

This table shows the results of the multilinear regression (13) for the panel regression (fixed-effects), considering only currency pairs where the average interest differential is smaller than 1%, is at least 1%, and is at least 2%, respectively, using the monthly 10-year swap return (β_{Return}) and the 1-month change of the implied volatility ($\beta_{Volatility}$) as the explanatory variables, and the 1-month change of the exchange rate as the explained variable. The period is from 1993-2012. The reported t-statistics are Newey-West estimators.

The higher the interest differential, the higher the beta of the change of volatility. The R-squared increases from 0.10 to 0.15 for interest differentials that are at least 1%, and to 0.19 for differentials that are at least 2%. However, the R-squared falls to 0.03 when the interest differential is smaller than 1%, since the effect of the change of volatility decreases. The β_{Return} does not change significantly with different levels of interest differentials.

Table 9 presents the results of multilinear regression (13), comparing monthly, weekly, and daily data.

Table 9: Multilinear Regression with Different Return Periods 1993-2012

		R2	β_{Return}		$\beta_{Volatility}$		Observations
Panel	Monthly	0.10	0.30	(7.81)	0.75	(10.04)	5‘040
Panel	Weekly	0.09	0.20	(10.28)	0.72	(12.86)	21‘903
Panel	Daily (avg[5])	0.05	0.11	(3.77)	0.51	(6.95)	105‘798

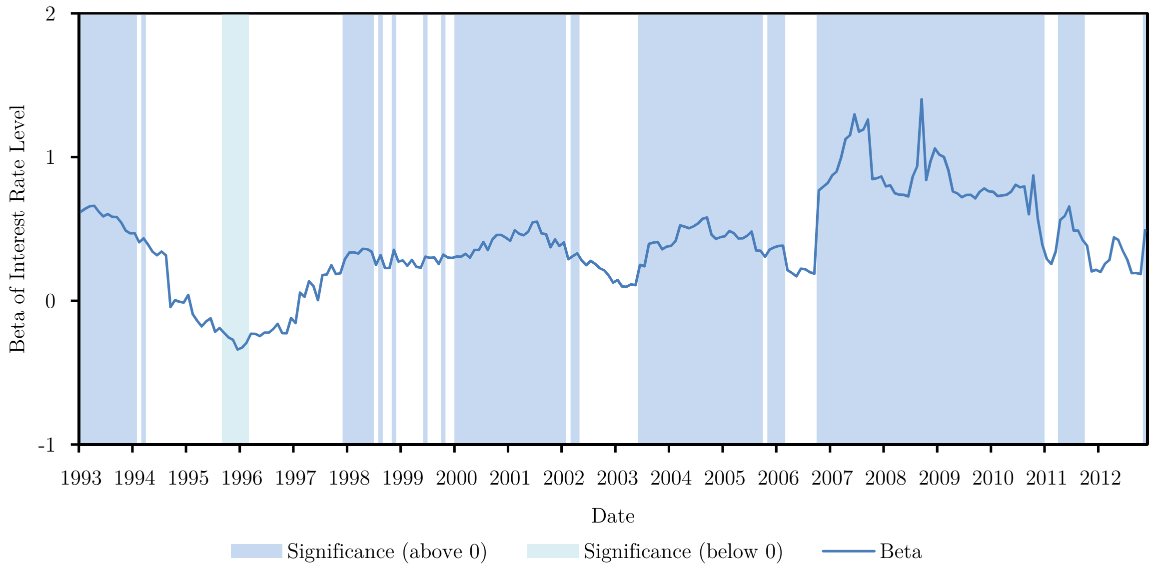
This table shows the results of multilinear regression (13) for the panel regression (fixed-effects) of all currency pairs, using different return periods (monthly, weekly, and daily) for the 10-year swap return (β_{Return}) and the change of the implied volatility ($\beta_{Volatility}$). The period is from 1993-2012. The reported t-statistics are Newey-West estimators.

Both the swap return difference and the change of volatility are also significant at shorter horizons, although the R-squared decreases. One problem with shorter data is that the interest rates and the exchange rates are not recorded at exactly the same time.

[5] For the daily data, I used the average data of each individual regression, since the capacity in Matlab is too limited to conduct a regression with three variables and roughly 100’000 observations per variable.

Figure 13 presents the 2-year rolling panel regression of the swap returns, considering only USD currency pairs. Although the beta is lower than it is in the total sample, it is still often significantly larger than 0.

Figure 13: 2-Year Rolling Regression 10-Year Swap Return Difference for USD Currency Pairs



This figure shows the beta of the 2-year rolling panel regression for USD currency pairs (no fixed-effects as the horizon is too short), using the monthly 10-year swap return as the explanatory and the 1-month change of the exchange rate as the explained variable for the period from 1991-2012. The shaded areas are the periods in which the beta is significantly different from zero at the 95% significance level. A 2-year rolling regression is regression (8) for 24 months, which means the first point for the 1-month regression is the beta over the period from January 1991 to December 1992, and the last point is the beta over the period from January 2011 to December 2012. The significance level is measured with Newey-West estimators.

Table 10 presents the results of the multiple regression of the 10-year swap return difference of month t and the lagged value t-1, as well as the change to the volatility.

Table 10: Multilinear Regression with Lagged Variables 1993-2012

	R2	$\boldsymbol{\beta_{Return}(t)}$	$\boldsymbol{\beta_{Return}(t-1)}$	$\boldsymbol{\beta_{Volatility}(t)}$	$\boldsymbol{\beta_{Volatility}(t-1)}$	Observations
Panel	0.11	0.31 (7.95)	0.09 (3.00)	0.77 (9.77)	0.20 (3.03)	5019

This table shows the results of multilinear regression (13) for the panel regression (fixed-effects) of all currency pairs, including the 1-month lagged values of β_{Return} and $\beta_{Volatility}$ as the explanatory variables. The period is from 1993-2012. The reported t-statistics are Newey-West estimators.

The regression results suggest that the 10-year swap return difference and the change of the implied volatility affect not only the current exchange rate, but also the exchange rate of the next month, although there is no autocorrelation in the swap return or in the change of the implied volatility, as the Durbin-Watson tests reveals. However, exchange rates have unit roots and are autocorrelated, which is probably the reason for this result. If I use only the lagged variables, the R-squared is reduced to 0.00, but the values of the lagged variables are similar, at 0.08 for β_{Return} and 0.14 for $\beta_{Volatility}$, respectively.

The results are also robust for the choice of the interest rate maturity. These panel regression results are presented in Table 11, and the 2-year rolling regression of selected maturities is presented in Figure 14.

Table 11: Interest Rate Return Difference with Different Interest Rate Maturities

Interest Rate	Start	End	R2	$\boldsymbol{\beta_{Return}}$[6]	
1M	31.12.1990	31.12.2012	0.00	-0.83	(-2.09)
2M	31.12.1990	31.12.2012	0.00	-0.53	(-1.24)
3M	31.12.1990	31.12.2012	0.00	-0.21	(-0.48)
6M	31.12.1990	31.12.2012	0.00	0.89	(1.98)
12M	31.12.1990	31.12.2012	0.02	1.06	(4.40)
2Y	31.12.1990	31.12.2012	0.04	1.04	(7.21)
3Y	31.12.1990	31.12.2012	0.05	0.84	(8.42)
4Y	31.12.1990	31.12.2012	0.05	0.73	(9.14)
5Y	31.12.1990	31.12.2012	0.06	0.64	(9.43)
7Y	31.12.1990	31.12.2012	0.05	0.51	(9.96)
10Y	31.12.1990	31.12.2012	0.03	0.34	(8.98)
1M	31.12.2001	31.12.2012	0.00	1.79	(2.46)
12M	31.12.2001	31.12.2012	0.06	2.58	(7.61)
5Y	31.12.2001	31.12.2012	0.20	1.61	(16.47)
10Y	31.12.2001	31.12.2012	0.12	0.80	(13.82)
15Y	31.12.2001	31.12.2012	0.07	0.45	(11.28)
20Y	31.12.2001	31.12.2012	0.06	0.33	(10.02)
30Y	31.12.2001	31.12.2012	0.03	0.19	(7.87)

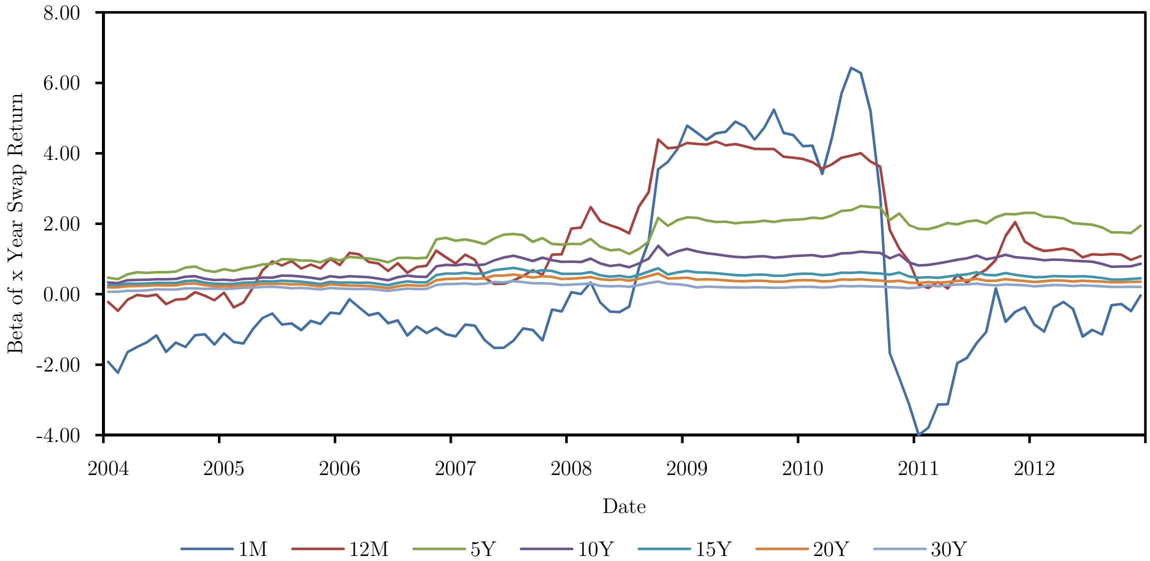
This table shows the results of panel regression (8) (fixed-effects) for different interest rate maturities, using the monthly swap return as the explanatory and the 1-month change of the exchange rate as the explained variable in the period from 1991-2012. The reported t-statistics are Newey-West estimators.

For short-term maturities of up to three months, the β_{Return} is negative. This is because it is dominated by the β_{Level}. For one month, the β_{Return} is the same as the β_{Level}. For longer maturities, the change to the interest rate increasingly dominates the monthly return. However, the level of the β_{Return} seems to be confusing at first glance, as it has its maximum at the 12-month interest rate, and its highest R-squared at the 5-year interest rate. The reason for this is that a small interest rate change has much greater return implications for long-term interest rates. Thus, the long-term interest rate returns typically have a higher magnitude than do

[6] The 30-year swap rates start in 2002.

medium-term interest rate returns. As we regress those returns against the same magnitude of exchange rate returns, the β_{Return} is smaller for long-term interest rates than it is for medium-term interest rates.

Figure 14: 2-Year Rolling Regression with Different Interest Rate Maturities



This figure shows the beta of the 2-year rolling panel regression (no fixed-effects as the horizon is too short), using monthly swap returns with different maturities as the explanatory and the 1-month change of the exchange rate as the explained variable in the period from 2002-2012. A 2-year rolling regression is regression (8) for 24 months, which means that the first point for the 1-month regression is the beta over the period from January 2002 to December 2003, and the last point is the beta over the period from January 2011 to December 2012. The significance level is measured with Newey-West estimators.

These results can be explained in the context of expectation theory. If expectation theory worked perfectly and all investors knew the development of future interest rates, the return of a short-term bond would be equal to the return of a long-term bond. In this case, the β_{Return} would be the same for all maturities. However, future interest rates are unknown, except for the 1-month interest rate in our regression, as the maturity has the same length as the regression window. For all longer maturities, there is uncertainty regarding future interest rate development. We can divide the return of each maturity into an expected and an unexpected return. The expected return equals the 1-month interest rate.

The unexpected return is the difference between the return of the bond and the 1-month interest rate. We can regress the expected return against the 10-year swap return, giving us an R-squared of 0.00. However, regressing the expected return on the 12-month returns gives an R-squared of 0.30; thus, the 12-month return can be partly explained by the expected return. On the other hand, the regression of the unexpected return against the 10-year return results in an R-squared of 0.99. For the 12-month return, the R-squared is 0.17. Therefore, almost everything in the 10-year return is explained by the unexpected return, while the 1-month interest rate has no explanatory power for the 10-year return. The 12-month return is partly explained by the expected return and the unexpected return.

The result is similar when the expected and the unexpected returns are regressed against the exchange rate change. The unexpected return (regression 11) has an R-squared of 0.04 and a beta of 0.35, similar to if we were to regress the 10-year return against the exchange rate change. The expected return has an R-squared of 0.00 and a beta of -0.83. This means that the exchange rate is driven by the unexpected interest rate return, which is determined by unexpected interest rate changes. As long-term interest rate returns are dominated by unexpected returns and short-term interest rate returns are dominated by expected exchange rate changes, this explains why UIP works well for long-term interest rates, and reasonably well for medium term interest rates, but not for short-term interest rates.

This proves that short-term and long-term interest rate return differentials have a very loose relationship, and helps to explain why the β_{Return} is in line with UIP for medium and long-term interest rates, but not for short ones. If one thing is certain, it is that it cannot work for both equally, unless the short-term and long-term interest rate return differentials are equal.

I also tested a regression with only the change of the interest differential, where the beta is -1.72, which is as expected. An increase in the foreign interest rate leads to an appreciation of the foreign currency. The increase in the foreign interest rate also leads to a negative swap return, and that is why the β_{Return} is positive.

II.3.9 Econometrics

It should be stressed that, as the β_{Level} is close to -1, there is a second order effect that would implicitly result in a positive β_{Return} . However, the effect on the β_{Return} is only about 0.01. The influence depends on the interest rates, and is between 0.008 and 0.012 with interest rates between 0 and 10%. As an example, let us assume that interest rates are 0 for both currencies. The foreign interest rate then increases to 1%. With a β_{Level} of -1 in UIP, the foreign currency will appreciate $\frac{1}{12}\%$ (as the interest rate difference is now -1%), which is 0.0833%. However, the present value of the 10-year foreign bond decreases to 9.87%, which means the interest return difference is +9.87%. Thus, if we regress the interest rate return of 9.87% on an exchange rate return of 0.0833%, the β_{Return} would be roughly $0.01(=\frac{0.0833}{9.87}\%)$.

The results can be misleading if the variables have a unit root or are co-integrated. Table 12 presents the unit root test for exchange rates, exchange rate returns, interest differentials, interest returns, and differentials. I used the augmented Dickey-Fuller test for the unit root.

Table 12: Unit Root Test 1991-2012

		Exchange Rate Level	Exchange Rate Return	Swap Differentials Level	Swap Return Difference
USD	JPY	Yes	No	Yes	No
USD	GBP	Yes	No	Yes	No
USD	CHF	Yes	No	No	No
USD	CAD	Yes	No	Yes	No
USD	DEM	Yes	No	No	No
JPY	GBP	Yes	No	No	No
JPY	CHF	Yes	No	Yes	No
JPY	CAD	Yes	No	Yes	No
JPY	DEM	Yes	No	Yes	No
GBP	CHF	Yes	No	Yes	No
GBP	CAD	Yes	No	Yes	No
GBP	DEM	Yes	No	Yes	No
CHF	CAD	Yes	No	Yes	No
CHF	DEM	Yes	No	Yes	No
CAD	DEM	Yes	No	Yes	No
USD	AUD	Yes	No	Yes	No
AUD	JPY	Yes	No	No	No
AUD	GBP	Yes	No	No	No
AUD	CHF	Yes	No	Yes	No
AUD	CAD	Yes	No	Yes	No
AUD	DEM	Yes	No	No	No

This table shows the augmented Dickey-Fuller unit root test for the exchange rates, exchange rate returns, swap differentials, and swap differential returns for all currency pairs. 'Yes' means that a unit root cannot be rejected, while 'no' means that a unit root can be rejected.

The unit root test shows that the exchange rate has a unit root. The interest differential has a unit root in 15 of 21 currency pairs. The implied volatility level also has a unit root (not reported in the table). However, using the deltas of the variables instead of the level, the unit root problem vanishes for all variables. Thus, results from regression (2) with the β_{Level} might be misleading as the interest rate levels are used, while there are no such problems with β_{Return} and $\beta_{\text{Volatility}}$.

Table 13 presents the results for the Johansen cointegration test for cointegration. The cointegration tests shows that the exchange rate and the level of the interest differentials are cointegrated in 11 of 21 currency pairs. The exchange rate and the level of volatility are also cointegrated in 9 of 21 currency pairs, due to the endogenic problems. Even exchange rate returns and the level of interest differentials are cointegrated for three currency pairs. However, interest differential returns and volatility changes are not cointegrated with the exchange rate return.

Table 13: Cointegration Test 1993-2012

		Exchange Rate Level– Swap Differentials Level	Exchange Rate Level - Volatility Level	Exchange Rate Return – Swap Differentials Level	Exchange Rate Return - Swap Return Difference	Exchange Rate Return – Volatility Change
USD	AUD	Yes	Yes	No	No	No
USD	JPY	Yes	Yes	Yes	No	No
USD	GBP	No	No	No	No	No
USD	CHF	Yes	Yes	No	No	No
USD	CAD	Yes	Yes	No	No	No
USD	DEM	No	No	No	No	No
AUD	JPY	No	No	No	No	No
AUD	GBP	Yes	Yes	No	No	No
AUD	CHF	No	No	No	No	No
AUD	CAD	No	No	No	No	No
AUD	DEM	No	No	No	No	No
JPY	GBP	Yes	No	Yes	No	No
JPY	CHF	No	No	No	No	No
JPY	CAD	Yes	No	No	No	No
JPY	DEM	No	No	Yes	No	No
GBP	CHF	Yes	Yes	No	No	No
GBP	CAD	Yes	Yes	No	No	No
GBP	DEM	Yes	Yes	No	No	No
CHF	CAD	No	No	No	No	No
CHF	DEM	Yes	Yes	No	No	No
CAD	DEM	No	No	No	No	No

This table shows the cointegration test for exchange rates, exchange rate returns, swap differentials, swap differential returns, volatility levels, and volatility changes for all currency pairs. 'Yes' means that the cointegration of rank 1 cannot be rejected, while 'no' means the cointegration rank 1 can be rejected.

Table 14 presents the results of the Granger causality test for interest differential returns, exchange rate returns, and volatility changes.

Exchange rate returns do not predict interest differential returns or volatility changes (except for USD-CAD), but interest differential returns and volatility changes do sometimes predict exchange rate returns. This confirms that exchange rate returns must be on the left side of the regression.

Table 14: Granger Causality Test 1991-2012

		Swap Return Difference - Exchange Rate Return	Volatility Change-Exchange Rate Return	Exchange Rate Return -Swap Return Difference	Exchange Rate Return-Volatility Change
USD	AUD	No	Yes	No	No
USD	JPY	No	No	No	No
USD	GBP	No	No	No	No
USD	CHF	No	Yes	No	No
USD	CAD	No	Yes	No	Yes
USD	DEM	No	No	No	No
AUD	JPY	Yes	No	No	No
AUD	GBP	Yes	No	No	No
AUD	CHF	Yes	No	No	No
AUD	CAD	Yes	No	No	No
AUD	DEM	Yes	No	No	No
JPY	GBP	No	No	No	No
JPY	CHF	No	Yes	No	No
JPY	CAD	No	Yes	No	No
JPY	DEM	No	Yes	No	No
GBP	CHF	No	No	No	No
GBP	CAD	Yes	No	No	No
GBP	DEM	No	No	No	No
CHF	CAD	Yes	No	No	No
CHF	DEM	Yes	No	No	No
CAD	DEM	Yes	No	No	No

This table shows the Granger causality test for exchange rate returns, swap differential returns, and volatility changes for all currency pairs. 'Yes' means variable 1 causes variable two, while 'No' means that it does not.

Table 15 presents the results of a vector autoregression for exchange rate returns and a swap return difference with three lags.

The vector autoregression does not show a clear pattern for exchange rate returns and swap return difference individually; thus, we can conclude that exchange rate returns have no autocorrelation, and neither have swap returns. However, the first lag of the swap return difference has a positive beta for most currencies, which confirms the results in Table 10 with lagged variables. Half of the currency pairs are also positive for t-2.

Table 15: Vector Autoregression 1991-2012

		Exchange Rate Returns on Exchange Rate Returns			Swap Return Difference on Exchange Rate Returns			Swap Return Difference on Swap Return Difference			Exchange Rate Returns on Swap Return Difference		
		Beta t-1	Beta t-2	Beta t-3	Beta t-1	Beta t-2	Beta t-3	Beta t-1	Beta t-2	Beta t-3	Beta t-1	Beta t-2	Beta t-3
USD	AUD	0.03	-0.05	0.15	0.20	0.17	0.15	-0.02	0.01	0.00	-0.01	-0.04	0.04
USD	JPY	0.00	0.09	0.02	0.13	-0.06	-0.17	-0.03	0.10	-0.01	-0.07	-0.25	-0.08
USD	GBP	0.13	-0.03	0.14	-0.06	-0.08	-0.17	0.08	-0.03	0.01	-0.14	-0.14	-0.07
USD	CHF	-0.02	-0.08	0.06	0.16	0.28	0.03	0.00	-0.02	0.04	-0.06	-0.06	-0.05
USD	CAD	-0.05	0.02	0.01	0.09	-0.09	0.13	-0.02	-0.04	0.02	-0.11	-0.03	-0.04
USD	DEM	0.04	-0.06	0.10	0.17	0.08	-0.02	-0.01	-0.07	0.01	-0.04	-0.04	-0.02
AUD	JPY	0.02	0.05	-0.04	0.27	0.07	0.05	-0.02	0.08	-0.04	0.06	-0.13	0.03
AUD	GBP	-0.09	-0.16	0.01	0.39	0.22	0.00	0.01	0.02	-0.00	-0.14	-0.11	-0.01
AUD	CHF	-0.03	-0.02	-0.00	0.27	0.42	0.01	-0.03	0.05	-0.06	0.00	-0.00	0.01
AUD	CAD	-0.09	-0.12	0.11	0.31	0.10	0.13	0.04	0.01	0.09	-0.15	0.01	-0.05
AUD	DEM	-0.02	-0.06	0.02	0.28	0.38	0.11	-0.06	0.03	-0.07	0.02	-0.03	0.10
JPY	GBP	0.08	0.11	0.05	0.06	-0.22	-0.14	0.02	0.04	0.02	-0.08	-0.17	-0.08
JPY	CHF	-0.01	-0.02	0.08	-0.05	0.36	-0.09	-0.02	0.00	0.02	-0.08	0.04	-0.03
JPY	CAD	-0.02	0.10	0.00	0.14	-0.13	-0.06	0.02	0.07	-0.01	-0.10	-0.13	-0.04
JPY	DEM	0.03	0.05	0.02	-0.06	0.03	-0.03	-0.03	0.05	-0.02	-0.02	-0.18	0.04
GBP	CHF	0.00	0.04	0.03	-0.10	0.10	0.07	-0.05	-0.00	0.02	-0.04	-0.03	-0.16
GBP	CAD	-0.09	-0.06	0.07	0.32	-0.13	-0.08	0.07	0.05	0.06	-0.15	-0.23	-0.17
GBP	DEM	0.04	-0.06	0.10	-0.09	-0.18	0.12	-0.01	-0.07	0.00	-0.07	-0.12	-0.08
CHF	CAD	-0.09	0.07	-0.00	0.29	0.14	-0.05	-0.02	0.03	0.01	-0.07	-0.02	-0.01
CHF	DEM	-0.11	0.03	0.01	-0.13	-0.05	-0.01	-0.01	0.06	-0.04	-0.17	-0.04	-0.13
CAD	DEM	-0.03	-0.01	0.04	0.31	-0.05	0.11	-0.02	0.01	0.01	-0.07	-0.06	-0.03

This table shows the betas of the vector autoregression for exchange rate returns and swap differential returns for all currency pairs with three lags.

II.4 Concluding Remarks

The uncovered interest rate parity has been a puzzle for decades, particularly when confronted with empirical data. The analysis has been incomplete, as only short-term interest rates have been analyzed and risk aversion has been ignored. This puzzle can be solved, to a degree, by taking long-term interest rate returns or changes in long-term bond prices, respectively. The time-varying risk plays an important part in the UIP regression. However, the important thing is to take the return of the long-term interest rate - here, I used the 10-year swap return. The return also includes unexpected interest rate returns and therefore avoids the problem of future interest rate estimation errors. In the empirical analysis, I found a beta that had been pushed close to 1 in the past couple of years, confirming the UIP theory. Thus, the failure of UIP for short-term maturities is caused by the failure of the expectation theory of the interest rate term structure. As a result, if a model to forecast future long-term interest rates were to exist, this model should also be able to forecast future spot exchange rates, something that hardly any model has ever been able to do.

In terms of further research, it would be interesting to analyze the constituents of the interest rates. Generally, interest rates consist of the risk-free rate, and a liquidity premium plus a credit risk premium. So far, the G7 countries have been considered to be very liquid and low-risk; thus, the swap rates should represent the riskless rate. However, this might no longer be true, as the market risk of the G7 countries is no longer considered to be identical by the financial markets. In the latter case, an increase in the interest rate driven by increased credit risk will not lead to an appreciation of that currency, but rather to a depreciation. Corte, Sarno, Schmeling, and Wagner [2013] have recently begun with this research.

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Part III

Synthetic International Equity Investment

Synthetic International Equity Investment

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Abstract

The first equity futures was launched in 1982 on the S&P 500 Index. The futures market has grown enormously since then, and is more liquid than is the cash market in many countries. The analysis here proves that the futures market is highly efficient from many perspectives. Transaction costs in futures are on average one third of those in the cash market. Arbitrage is hardly ever possible. As futures contain information about future dividends, they can be used to analyze the market estimation for dividends. Even this proves to be quite efficient, with an exception in 2008, when the market either underestimated the subsequent dividends, or the increased risk aversion caused market values of future dividends that were too low. Withholding taxes caused a split between the futures and the cash markets. For investors who cannot reclaim foreign withholding taxes, it is beneficial for them to invest only in futures, as dividend earnings implied in the futures price are not taxed.

III.1 Introduction

More than 30 years ago, on April 21 in 1982, the S&P 500 Index Futures was launched as one of the first equity index futures.[1]

The growth of equity futures trading continued in the following years; thus, the major stock indices of almost every country currently have a futures market. This offers the opportunity to invest in a global equity portfolio using only a few futures (synthetic investments), instead of thousands of single stocks (cash investments).

The futures market offers numerous advantages compared to the cash market. The transaction costs are much lower in the (established) futures market, and the liquidity is currently two times higher (compared to chapter III.2.4). The improved liquidity enables larger trades without market impact. Furthermore, a single futures trade generates the exposure to several hundred individual stocks at once. Unproblematic leverage is also possible, since the margin requirements for futures are roughly 10% those of the nominal, and short sales are possible without additional costs. A little-known advantage of futures is the implied withholding tax benefit: While a foreign investor often loses the withholding tax on dividend earnings to some extent, the future, theoretically, has no withholding tax losses. Trading in some futures is possible 24 hours a day, which is especially important for global portfolios, while cash markets are open asynchronously.

It is commonly known that short-term investors should buy futures, since they are cheaper in terms of transaction costs, while long-term investors should buy cash stocks, as ongoing costs of the cash markets are lower. However, it is unclear how long an investment should last in order for cash investments to be cheaper than futures.

Unfortunately, despite the long and broad acceptance of MSCI World as a global benchmark, there is still no liquid MSCI World futures available. In recent years, the MSCI EAFE Futures (Europe, Australia, and the Far East) have achieved acceptance, but the daily turnover is only $150 million, which is still very small. As an example, the EURO STOXX 50 futures have a daily turnover of $44 billion. Thus, a global futures investment for large investors has to be replicated using a combination of several stock index futures.

Jorion and Roisenberg (1993) were the first to analyze the performance and transaction costs of an international, synthetic portfolio. They used five futures to replicate the MSCI World

[1] The Value Line Index Futures traded on the Kansas City Board of Trade was the first equity futures, launched on February 16, 1982. This futures was far less successful than the S&P 500 Futures, and is rarely still traded (Source http://www.cftc.gov/About/HistoryoftheCFTC/history_1980s.html).

international equity benchmark for the period from 1987-1990. At this time, one-way transaction costs were 15 bps in the cash US stock market and 4 bps in the futures market. MSCI World contained 21 countries, although South Africa has dropped out, and Austria, Greece, Israel, and Portugal have been included in the index in the meantime. The tracking error of these five futures, compared to the MSCI World Index, was 3.02%, and the outperformance of the futures was 4.4%.

Waring and Attwood (1999) analyzed the horizon from 1989-1996 for nine country indices of the MSCI EAFE Benchmark. The US and Canada were excluded, since these countries are domestic and not international from the perspective of a US investor. Their study focuses on a detailed transaction cost comparison between futures and cash investments, including commissions, market impact, taxes, management fees, securities lending revenues, rollover costs, and so-called futures mispricing costs. They estimate the net costs to establish and maintain positions for one year to be within a range of between -0.05% and -1.24% in the cash market, and 0.86% and -1.45% in the futures market. Over a 10-year horizon, they found a positive return on the futures compared to the MSCI indices in the UK, France, Germany, and Italy, while the return was negative in Australia, Spain, Hong Kong, Sweden, and Japan. The tracking error of the futures, compared to the individual country indices, varied between 1 and 4%.

Hill and Naviwala [1999] analyzed the transaction costs, performance and tracking error of the S&P 500 for the period from 1992-1998. They estimated the annual costs for the S&P 500 futures to be 5-8, and 30 bps for the cash stocks. In their analysis, the futures performance was, on average, 5 bps below the gross return index. The standard deviation of the futures deviation from the fair value is denoted as 0.20%, and the tracking error between the futures and the index performance is 0.56% on a quarterly basis, and 0.98% on monthly basis. They mentioned that the annualized quarterly tracking error is a better measure, since the daily mispricing has a mean reversion.

In this paper, the analysis of the papers above is combined. In the first part, the theoretical considerations of the synthetic investments are analyzed. In the second part, the investments are analyzed empirically for individual futures compared to their index, while the last part presents the characteristics of a global portfolio with MSCI World as a benchmark. It will provide guidance for large international investors like pension funds, mutual funds, or ETFs who invest in global equity portfolios.

The individual analysis reveals that futures are priced very efficiently compared to their index in every way, with certain exceptions during the financial crisis. The price deviation of a futures to its index is usually within the no-arbitrage bounds, which are provided by the transaction costs. The transaction costs of futures are, on average, only 3.3 basis points, and therefore 1/3 of the cash market transaction costs. The return of a futures is similar to the return of a cash investment. Even the cost to rollover a futures has been quite exact, although the dividends are not precisely known in advance. It was only during the financial crisis that the future dividends were either underestimated, or were priced very low due to risk aversion.

A global futures portfolio has a tracking error of 0.67% to the MSCI World index. The futures return was 0.29% below the total gross return of a cash investment, but was 0.37% above the total net return of a cash investment. The difference between the total return and the total net return of a cash investment is the withholding taxes. Thus, for long-term investments, the tax status of the investors is decisive. If they are not able to reclaim withholding taxes in foreign countries, they will have a higher return with a futures investment in all cases. For investors who can reclaim withholding taxes, the holding period for futures should not exceed several months, because otherwise the ongoing costs will exceed the initial transaction cost benefit.

III.2 Framework for Synthetic Equity Investment

III.2.1 Data

The basic universe is the MSCI World Index of the developed markets. Table 1 presents the country universe, their weights in the MSCI World developed as of December 31 2012, and the related futures. If there were more than one futures for a country with different indices, I chose the futures whose index had the lowest tracking error to the MSCI country index. MSCI uses the market capitalization of the MSCI Country Indices to determine the weights of each country in the MSCI World. There are currently no futures available for Ireland and New Zealand. For Japan, I used the TOPIX futures instead of the more famous Nikkei 225 Future, since the TOPIX futures has a smaller tracking error to MSCI Japan.[2]

The countries in the MSCI Eurozone (France, Germany, Spain, Italy, the Netherlands, Finland, Belgium, Austria, Ireland, Greece, and Portugal) can alternatively be replicated with the EURO STOXX 50 Futures. MSCI Eurozone has a weight of 12.33% in the MSCI World Index.

[2] The TOPIX Futures has roughly 1600 constituents, while the MSCI Japan has only 350. However, the TOPIX and the MSCI Japan are both capital-weighted indices, while the Nikkei is a price-weighted index.

Table 1: Universe

MSCI	Weight	Future
MSCI USA	52.56%	S&P 500 Futures
MSCI Japan	8.55%	TOPIX Futures
MSCI UK	9.65%	FTSE 100 Futures
MSCI Canada	4.89%	S&P TSX 60 Futures
MSCI France	4.09%	Cac40 Futures
MSCI Australia	3.81%	S&P ASX 200 Futures
MSCI Switzerland	3.56%	SMI Futures
MSCI Germany	3.75%	DAX 30 Futures
MSCI Spain	1.29%	IBEX 35 Futures
MSCI Sweden	1.35%	OMXS30 Futures
MSCI Italy	0.96%	FTSE MIB Futures
MSCI Netherlands	1.07%	AEX Futures
MSCI Hong Kong	1.34%	Hang Seng Futures
MSCI Singapore	0.80%	MSCI Singapore Futures
MSCI Finland	0.34%	OMXH25 Futures
MSCI Denmark	0.49%	OMXC20 Futures
MSCI Belgium	0.50%	BEL20 Futures
MSCI Israel	0.23%	TA 25 Futures
MSCI Norway	0.39%	OBX Futures
MSCI Austria	0.13%	ATX Futures
MSCI Ireland	0.11%	
MSCI Greece	0.02%	FTSE ATHEX 20 Futures
MSCI Portugal	0.08%	PSI 20 Futures
MSCI New Zealand	0.05%	

This table presents the universe of the MSCI World with a due date of December 31 2012, the weight of each country and the corresponding future.

III.2.2 Data Source

The primary data source is Bloomberg, since it has an extremely comprehensive database of futures. The data horizon is from February 26 1999 to December 31 2012. There are two reasons why I did not look at older data. Firstly, the completeness of available data decreases rapidly in the earlier years. Secondly, older data are less representative for trading advice today.

Intraday Data: For the bid-ask spread and trading volume analysis, I used the intraday tick-by-tick data at Bloomberg from September 1 2009 to February 28 2010. For the intraday data of the S&P 500 in 2008, the data provider is also Bloomberg.

MSCI: The MSCI indices are the official international indices in USD, including both gross and net return indices.

FX: For the currency exchange rate, the official MSCI exchange rates against the USD were used.

Interest Rates: The maturity of the interest rate is between one day and six months. If the maturity does not match the expiration, the interest rate is linearly interpolated. The LIBOR and swap interest rates are used. As the swap rate is the preferred rate, but is not available for the beginning of the time period, LIBOR is used in the first period and swap rates in the second. Table 2 indicates which rate is used, and at which date it switches from LIBOR to swap rates.

Table 2: Interest Rates

	First Period	Second Period	Change Date
USD	LIBOR	Swap Rate	04.12.2001
JPY	LIBOR	Swap Rate	15.03.2002
GBP	LIBOR	Swap Rate	14.12.2000
EUR	Swap Rate	Swap Rate	-
CAD	LIBOR	Swap Rate	03.05.2002
AUD	LIBOR	Swap Rate	23.10.2001
CHF	LIBOR	Swap Rate	17.08.2000
SEK	STIBOR	Swap Rate	03.08.2004
HKD	HIBOR	Swap Rate	08.08.2001
SGD	Swap Rate	Swap Rate	07.11.2001
DKK	CIBOR	Swap Rate	01.02.2001
NOK	NIBOR	NIBOR	-

This table presents the chosen interest rate for each currency, and the switch date from LIBOR to the swap rate.

Futures/Indices: Not all futures or gross return indices exist for the entire period. Table 3 shows the start date of the futures/indices that began during the period in question.

Table 3: Start Date Futures/Indices

Future/Index	Start Date
S&P ASX 200	31.05.2000
FTSE MIB	31.03.2004
OMXS30	28.02.2005
FTSE Athex 20	28.11.2008
OMXH25	30.09.2005
EURO STOXX 50	31.01.2001

This table presents the start date of futures and indices for all countries in which one of them started later than the general starting period of February 26 1999.

The EURO STOXX 50 Futures is available for the entire period, but the EURO STOXX 50 gross return index is only available from January 31 2001. Comparisons between the return of the EURO STOXX 50 Futures and the gross return index start on January 31 2001, while portfolio comparisons between futures and the MSCI also start on December 31 1999, since all MSCI data are available from the beginning.

The following gross return indices are from Datastream, since the data quality is either insufficient or the data are not available from Bloomberg: S&P TSX 60, S&P ASX 200, AEX, Hang Seng, MSCI Singapore, OMX Copenhagen, FTSE Athex 20, and OMX Helsinki 25.

Transaction costs: The source for the brokerage costs is the Zuercher Kantonalbank (ZKB), based in Switzerland.

III.2.3 Transaction Costs

Transaction costs are an important factor in the performance of equity portfolios, particularly if the turnover within the portfolio is high. There are several sources of transaction costs. The most important are the spread of the bid and the offer price. A trade that must be executed immediately is traded at the bid price in the event of a sell-order, and at the offer price in the event of a buy-order. Furthermore, some countries demand a transaction fee, also called Tobin tax. Lastly, the bank that executes the order also charges a brokerage fee, which can be substantially higher than all the fees above. The brokerage fee varies depending on the bank, the exchange, the client, and the order size. In this paper, I used the costs for institutional investors trading with the Swiss bank ZKB as a reference.

The spread to the mid-price is defined as follows:

$$½\, BA - Spread = \frac{P_a - P_b}{P_a + P_b}$$

where P_b is the bid price and P_a is the asking price. In the cash market, I took the weighted bid-ask spreads of each individual stock in the index to calculate the bid-ask spread of the index. For some futures and equity markets, bid and ask prices are not recorded, which is usually the case in open outcry markets without electronic trading. For those markets, I used the trade spread as a proxy for the spread to the mid-price. The methodology is from Wang, Chung, and Yang [2007].

$$½\, T - Spread = \frac{|TP_{t+1} - TP_t|}{TP_{t+1} + TP_t}$$

where T is the trade spread, which is the spread between the transaction price TP_t at time t

and the price TP_{t+1} at time t+1. TP_{t+1} is the next transaction price that is unequal to TP_t, and which had changed the direction of the price from increase to decrease, or vice versa. The brokerage fee for large program trades[3] in the MSCI World is 3 Bps at ZKB. This fee includes both the brokerage fee and the exchange fee, but not the taxes. The brokerage fee for futures is individual for each future, and is independent from the number of futures.

For futures, rollover costs incur. The brokerage fees for the rollover of a futures have to be paid twice, and a rollover spread also occurs. For most futures, there is actually a quoted bid- and ask-price for the rollover difference between the current and the next futures contract. This spread is typically lower than is the bid-ask spread. For these cases, I used the spread of the bid-ask rollover difference. If there a rollover spread available were not available, I took the ordinary bid-ask spread of the future.[4] Nonetheless, the rollover spread is an upper boundary for the actual costs, since there is no time pressure to execute the rollover immediately. Therefore, the rollover could be executed with a limited order, thus avoiding part- or full-spread costs.

Table 4 shows the transaction cost of the developed MSCI country indices and the corresponding futures.[5]

Table 4: Transaction Costs

Country	Instrument	Brokerage Fee	Transaction Tax	1/2 BA Spread	1/2 BA FX	Total	Rollover Costs
USA	S&P 500 Futures	0.2		1.2		1.3	2.2
USA	MSCI US	3.0	0.2	2.3		5.5	
Japan	TOPIX Futures	1.2		2.9	2.7	6.7	11.5
Japan	MSCI Japan	3.0		7.6	2.7	13.3	
UK	FTSE 100 Futures	0.4		0.7	1.8	2.9	5.5
UK[6]	MSCI UK	3.0	25.0	4.6	1.8	34.4	
France	Cac40 Futures	0.7		1.0	0.8	2.5	8.2
France	MSCI France	3.0		2.6	0.8	6.5	
Canada	S&P TSX 60 Futures	0.5		1.4	3.6	5.5	4.8
Canada	MSCI Canada	3.0		2.4	3.6	9.0	
Australia	S&P ASX 200 Futures	0.4		1.3	3.5	5.2	7.4

[3] According to the NYSE, a large program trade is a basket of at least 15 shares with a market value of $ 1 Mio. or more http://www.nyse.com/press/1294312649052.html.
[4] There is no rollover spread for FTSE MIB, OMXS30, or TA 25.
[5] The source for the brokerage fee is ZKB Asset Management. The sources for the bid-ask spread are the quoted spreads from September 1 2009 to February 28 2010, and constitute the average of all intraday quotes during the main trading sessions. For the rollover spreads, only the last trading week of the expiring futures was considered.
[6] The UK has a stamp duty of 50 bps for a buy per share and a 1£ levy for a sell per trade; thus, the tax is an average of 25 bps per trade.

Australia	MSCI Australia	3.0		5.2	3.5	11.7	
Germany	DAX 30 Futures	0.2		0.7	0.8	1.7	2.9
Germany	MSCI Germany	3.0		2.8	0.8	6.6	
Switzerland	SMI Futures	0.2		1.1	1.3	2.7	4.8
Switzerland	MSCI Switzerland	3.0		4.6	1.3	8.9	
Spain	IBEX 35 Futures	0.5		1.4	0.8	2.7	14.9
Spain	MSCI Spain	3.0		3.7	0.8	7.6	
Italy	FTSE MIB Futures	0.4		1.5	0.8	2.7	15.3
Italy	MSCI Italy	3.0		4.5	0.8	8.3	
Netherlands	AEX Futures	0.6		1.2	0.8	2.6	22.7
Netherlands	MSCI Netherlands	3.0		6.9	0.8	10.7	
Sweden	OMXS30 Futures	4.5		1.9	4.6	11.0	153.6
Sweden	MSCI Sweden	3.0		4.9	4.6	12.5	
Hong Kong	Hang Seng Futures	0.5		0.6	1.8	2.8	15.4
Hong Kong	MSCI Hong Kong	3.0	10.9	6.7	1.8	22.4	
Singapore	MSCI Singapore Futures	2.3		2.3	3.3	7.8	71.4
Singapore	MSCI Singapore	3.0		13.3	3.3	19.6	
Belgium	BEL20 Futures	2.1		13.3	0.8	16.2	57.3
Belgium	MSCI Belgium	3.0		4.0	0.8	7.8	
Denmark	OMXC20 Futures	9.6		20.0	2.2	31.7	503.4
Denmark	MSCI Denmark	3.0		7.9	2.2	13.0	
Norway	OBX Futures	7.6		9.0	5.0	21.6	306.0
Norway	MSCI Norway	3.0		4.6	5.0	12.6	
Greece	FTSE Athex 20 Futures	2.1		6.2	0.8	9.2	39.3
Greece	MSCI Greece	3.0		29.6	0.8	33.5	
Austria	ATX Futures	2.7		21.9	0.8	25.4	108.4
Austria	MSCI Austria	3.0		7.9	0.8	11.8	
Portugal	PSI 20 Futures	-		15.5	0.8	16.3	87.5
Portugal	MSCI Portugal	3.0		5.8	0.8	9.7	
Finland	OMXH25 Futures	-		18.7	0.8	19.5	32.0
Finland	MSCI Finland	3.0		5.1	0.8	8.9	
Ireland		-		-	0.8	0.8	
Ireland	MSCI Ireland	3.0	100.0	13.2	0.8	117.0	
New Zealand		-		-	4.8	4.8	
New Zealand	MSCI New Zealand	3.0		13.1	4.8	20.9	
Eurozone	EURO STOXX 50 Futures	0.7		1.8	0.8	3.3	12.1
Eurozone	MSCI Eurozone	3.0		4.2	0.8	8.0	
Israel	TA 25 Futures	-		10.5	8.3	18.8	84.1
Israel	MSCI Israel						

This table presents the transaction costs for futures and cash investments for each country in the MSCI world. The costs are denoted in basis points. The costs were recorded in 2010.

The lowest transaction costs are in the USA, with 1.3 bps for the futures and 5.5 bps for the cash market. The most expensive market is Ireland with 117 bps, due to its stamp duty of 100 bps, followed by the UK with 34.4 bps (25 bps stamp duty). In Belgium, Denmark, Norway, Austria, Portugal, and Finland, the futures are more expensive than is the cash market. The futures trading volume in those markets is below that of the cash volume.

Neal [1996] has a bid-ask spread of 64 bps for the S&P 500. Chan and Lakonishok [1993] estimated transaction costs for large institutional investors as being approximately 16 bps for S&P 500. Beebower and Priest [1980] estimated 70 bps transaction costs for the S&P500. Wang, Chung, and Yang [2007] estimated a bid-ask spread of 1.6 bps for the S&P 500 Futures. Kurov and Zabotina [2005] have 1.2 bps for S&P 500 Futures, which is already close to the 0.6 bps in this study. Kurov and Zabotina [2005] showed that the bid-ask spread depends strongly on the minimum tick size, which is currently 0.1 index points for the S&P 500. Thus, several factors led to lower transaction costs: liquidity increased during the past decades, the minimum tick size was reduced in many markets and the index level increased, all of which produced lower bid-ask spreads. The EURO STOXX 50 has a relatively high bid-ask spread, more than twice that of the Cac40 Futures. The reason is that the minimum tick size is high, with 1 index point, which equaled roughly 5 bps in 2012.

To my knowledge, Waring and Attwood [1999] were the only people who made a comparison of global transaction costs for futures and for the cash market. They analyzed nine countries in the MSCI EAFE, and found average transaction costs of 43 bps for the cash market, 31 bps for futures and 38 bps annual rollover costs. The problem is that they included market impact costs for a $1 billion portfolio, which is difficult to estimate. I have transaction costs of 13.7 bps for the cash market and 4.3 bps for futures in those countries, but I excluded market impact costs and included currency transaction costs. While I got 3 bps brokerage fees for the cash market and 0.9 Bps for futures, while they reported 12.1 for cash and 2.1 for futures.

The reasons that the transaction costs are so different are that the transaction costs have decreased during the last couple of decades, the brokerage fees of banks are different and are client specific, and some studies include market impact costs. However, the liquidity is currently very high, and the top level of both the S&P 500 and the EURO STOXX 50 order book have 50 million volume more than is tradable at the bid-ask price.

In this study, I excluded the market impact cost in order to obtain a proper comparison of transaction costs. It is reasonable to assume that, for a volume of several $ 100 million, there are no market impact costs, but only the bid-ask spread costs. Nonetheless, if the volume is higher than several $ 100 million, it might be reasonable not to trade everything at once or, if possible, to trade it in the closing auction.

III.2.4 Trading

Table 5 lists the number of stocks, the daily trading volume, and the trading hours of the MSCI country index compared to its corresponding future.

Table 5: Trading Details

Instrument	#Stocks	Volume[7]	Trading Hours[8]
S&P 500 Futures[9]	500	98'039	00:00-24:00
MSCI US	602	33'585	09:30-16:00
TOPIX Futures	1'600	4'196	19:00-05:00
MSCI Japan	344	14'936	19:00-01:00
FTSE 100 Futures	100	8'632	02:00-16:00
MSCI UK	102	6'733	03:00-11:30
Cac40 Futures	40	6'076	02:00-16:00
MSCI France	71	5'342	03:00-11:30
S&P TSX 60 Futures	66	1'573	06:00-16:00
MSCI Canada	99	3'790	09:30-16:00
S&P ASX 200 Futures	200	3'251	00:00-24:00
MSCI Australia	73	3'175	18:00-00:00
DAX 30 Futures	30	27'417	02:00-16:00
MSCI Germany	49	4'922	03:00-11:30
SMI Futures	20	2'277	02:00-16:00
MSCI Switzerland	40	3'084	03:00-11:30
IBEX 35 Futures	35	2'198	03:00-11:30
MSCI Spain	30	2'790	03:00-11:30
FTSE MIB Futures	40	2'183	03:00-11:30
MSCI Italy	34	3'638	03:00-11:30
AEX Futures	24	2'874	02:00-16:00
MSCI Netherlands	23	1'458	03:00-11:30
OMXS30 Futures	30	965	03:00-11:30
MSCI Sweden	30	1'440	03:00-11:30
Hang Seng Futures	42	9706	21:00-03:00
MSCI Hong Kong	40	815	21:00-03:00
MSCI Singapore Futures	29	364	19:30-12:00
MSCI Singapore	29	656	20:00-04:30
BEL20 Futures	20	7	03:00-11:30
MSCI Belgium	13	407	03:00-11:30

[7] The source for the trading volume is Bloomberg. The cash indices are the average trading volume of equities from September 1 2009 to March 24 2010. The futures consist of the median trading volume since there are spikes preceding the expiry of the futures, due to rollover transactions.
[8] Japan, Hong Kong, and Singapore have a break during local lunchtime. THE S&P ASX 200 Futures have 40 minutes' break after the closing of the cash market.
[9] The S&P 500 Futures have $5 billion trading volume, while the S&P 500 E-Mini Futures have $93 billion.

OMXC20 Futures	20	2	03:00-11:00
MSCI Denmark	13	398	03:00-11:30
OBX Futures	25	47	03:00-11:30
MSCI Norway	8	716	03:00-11:30
FTSE Athex 20 Futures	20	81	03:00-08:00
MSCI Greece	11	245	03:30-09:00
ATX Futures	20	6	03:00-11:30
MSCI Austria	8	161	03:00-11:30
PSI 20 Futures	20	1	03:00-11:30
MSCI Portugal	8	177	03:00-11:30
OMXH25 Futures	25	2	00:00-24:00
MSCI Finland	16	585	03:00-11:30
TA 25 Futures	25	164	03:00-11:30
MSCI Israel	17	367	02:30-11:00
EURO STOXX 50 Futures	50	43'873	02:00-16:00
MSCI Eurozone	268	19'802	03:00-11:30

This table presents the trading details for futures and cash investments for each country in the MSCI world. The number of stocks is the number of stocks in the future, or in the MSCI Index in June 2010.The volume is the daily trading volume in $ billion recorded during the period from September 1 2009 to March 24 2010. The trading hours are the periods in which trading is possible, as of June 30 2010.

Most futures match the MSCI index quite closely regarding the number of stocks. However, the TOPIX Futures has 1600 stocks, which is much more than that of the MSCI Japan, which has with 344. While the Nikkei 225 would seem to be more adequate considering the number of stocks, it is a price-weighted index and not a market-capitalization weighted index, like the MSCI Japan. Thus, the weights of the large stocks are more precise in the TOPIX future, an aspect that is more important since they are the main driver of the capital-weighted MSCI index. The EURO STOXX 50 Futures has only 50 stocks, compared to MSCI Eurozone with 268 stocks. For Singapore, there is an MSCI futures that is liquid.

Some countries have much greater trading volume in the futures than in the cash market. The S&P 500 Futures has the price-weighted volume of $98 billion per day, while the cash market has only $34 billion. Hill and Naviwala [1999] measured the volume of the S&P 500 in 1998, and found it was $34 billion. Thus, the volume has nearly tripled over the past 12 years. The EURO STOXX 50 Futures has $44 billion, while the cash market, which is even broader, has only $19 billion. The DAX Futures has a volume of $27 billion, compared to $ 5 billion in MSCI Germany, while the Hang Seng Futures are also much more liquid than are those of MSCI Hong Kong ($9 billion versus $1 billion). In Japan, the TOPIX Futures volume is, at $4 billion, much lower than that of the MSCI Japan ($15 billion). An equally large part of the Japanese futures market is traded on the Nikkei 225. In Belgium, Norway, Greece, Austria, Portugal, and Finland, the futures market is much more illiquid than is the cash market. The volume in these markets is below $100 million and, in Portugal, it is only $1 million. Hence, these futures are not practical for large futures portfolios.

In the major markets, the futures market is open for 14 hours per day, while the cash market is open for 8.5 hours. The S&P 500 and the S&P ASX 200 Futures are tradable 24 hours per day. In the smaller market, the trading hours for futures are similar to those of the cash market. However, the volume is much lower in the futures market outside of the cash market's trading hours. Bid-ask spreads are also higher, up to twice the regular value.

III.2.5 Withholding Taxes

Table 6 shows the withholding tax rate and the dividend yield of the MSCI country indices.

All countries except for the UK, Hong Kong, and Singapore deduct withholding taxes on dividends. International investors can lose those taxes. Most countries have double taxation treaties, which reduce the tax losses on dividends to 15% for most countries. An exception is the home country of the investors, where institutional investors do not pay withholding taxes. Thus, the market is fragmented: some investors pay full withholding taxes, some pay only reduced rates, and home investors do not pay any taxes at all. The MSCI contributes to this fact with two return indices: the total gross return index reinvests the full dividend amount, while the net return international index invests only the dividends' net full withholding taxes.[10]

Futures imply no explicit taxes, and there is no market fragmentation since the price is the same for all investors. The dividend return is included in the price. However, the taxes on dividends might be adjusted in the price, and interest earnings are taxable in the home country[11]. However, institutional investors can reclaim withholding taxes in the home country. Thus, the only aspect that is important for them is the futures implied dividend return compared to the dividend return that they would receive with cash equities.

The dividend yield is lowest in Japan with 1.7%, which is also the country that had the lowest interest rates during the past 10 years. New Zealand had the highest yield, with 6.7%. The average dividend yield in the MSCI World was 2.9%. An investor who has to pay full withholding taxes would have lost 0.66% per year if s/he had invested in the MSCI World. S/he could have reduced the loss to 0.35% under double taxation benefits.[12]

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[10] http://www.mscibarra.com/eqb/methodology/meth_docs/MSCI_Feb11_IndexCalcMethodology.pdf

[11] A synthetic investor can place the cash money completely in his home country and achieve interest earnings aboard with currency forwards or currency futures.

[12] In certain countries, such as the US, some investors like pension funds are even fully tax-exempt.

Table 6: Withholding Tax Rates

Country	Withholding Tax Rate	Dividend Yield	Full Withholding Tax Loss	Reduced Withholding Tax Loss
USA	30.00%	2.4%	0.72%	0.4%
Japan	7.00%	1.7%	0.12%	0.1%
UK	0.00%	3.9%	0.00%	0.0%
France	25.00%	3.5%	0.88%	0.5%
Canada	25.00%	3.7%	0.93%	0.6%
Australia	30.00%	4.9%	1.48%	0.7%
Germany	26.38%	4.0%	1.07%	0.6%
Switzerland	35.00%	2.1%	0.72%	0.3%
Spain	19.00%	4.1%	0.79%	0.6%
Italy	27.00%	5.3%	1.42%	0.8%
Netherlands	15.00%	4.1%	0.61%	0.6%
Sweden	30.00%	4.5%	1.34%	0.7%
Hong Kong	0.00%	4.2%	0.00%	0.0%
Singapore	0.00%	4.3%	0.00%	0.0%
Belgium	25.00%	4.1%	1.03%	0.6%
Denmark	28.00%	2.5%	0.70%	0.4%
Norway	25.00%	4.5%	1.12%	0.7%
Greece	10.00%	2.3%	0.23%	0.2%
Austria	25.00%	3.3%	0.82%	0.5%
Portugal	20.00%	4.2%	0.85%	0.6%
Finland	28.00%	5.0%	1.40%	0.7%
Ireland	20.00%	2.5%	0.50%	0.4%
New Zealand	15.00%	6.7%	1.00%	1.0%
Israel	20.00%	2.1%	0.00%	0.0%
Eurozone	25.00%	2.9%	0.00%	0.0%
World	23.14%	2.9%	0.65%	0.35%

This table shows the withholding tax rate and the dividend yield of the MSCI country indices. The dividend yield is from Datastream and reports the average yield from December 31 1999 to December 31 2009. The withholding tax rate is that of June 2010. The withholding tax loss is the dividend multiplied by the withholding tax rate. The reduced withholding tax loss is the dividend yield multiplied by the recued withholding tax rate under double tax agreements.

III.2.6 Theoretical Futures Price

A popular model to derive the fair value of the futures price is the cost-of-carry model. Kaldor [1939] described this model for commodity futures, which already have a much longer existence. For equity futures, the pricing formula is as follows (see, for example, Miller, Muthuswamy and Whaley [1994], or Neal [1996]):

$$F_t^* = I_t exp\left(r_t \frac{(T-t)}{365}\right) - \sum_{i=t+1}^{T} D_i exp\left(r_t \frac{(T-t-1)}{365}\right) \tag{1}$$

where $\mathrm{F_t^*}$ is the fair value of the futures price, $\mathrm{I_t}$ is the spot price of the index, $\mathrm{r_t}$ is the continuous interest rate with maturity T and is assumed to be constant over the contract life, T-t is the time to maturity, and $\mathrm{D_i}$ is the dividend paid on day i. From (1), it follows that $\mathrm{F_t^*} = \mathrm{I_t}$ on expiry. The fair value of the futures price is nothing other than the compounded value of the stock index at the expiry of the futures contract, less the compounded dividends until expiry. Thus, as the name says, it is the futures value of the current index price.

Futures contain almost no counterparty risk,[13] since the exchange guarantees the daily settlement of the contract. As a result, $\mathrm{r_t}$ is the risk-free interest rate. Currently, the OIS interest rate is considered to be the best proxy for the risk-free interest rate, since it is also settled daily via a cash settlement and there is no notional exchange.

Excursion: Risk-Free Interest Rates:

For the data from 1999 to 2002, I used the LIBOR as the interest rate, since the OIS is not yet available and the interbank risk was negligible during this period (compared to the interest rate analysis in the attachment). For data after 2002, I used the OIS because the OIS contains almost no interbank risk, while the LIBOR contains the full interbank risk spread. The LIBOR is the London interbank offered rate, which is the daily fixed offered interest rate of the British Bankers Association. The OIS is the overnight indexed swap, which has a fixed leg and a floating leg. The floating leg is tied to the daily overnight reference index. The fixed rate of this swap is probably the best risk-free interest rate that is available today (for a detailed argument, see Hull and White [2013]). Table 7 and Figure 1 present the descriptive statistics for the different types of US interest instruments.

The three-month LIBOR is, on average, 2.77% over the entire sample period, while the three-month treasury is 2.31. The 0.46% difference represents the credit risk. In the period between

[13] During the crash in October 1987 and the drop of the S&P 500 Futures of 29%, the default of the clearing house was not always impossible (see Bernanke [1990]).

2002-2006, this difference was much smaller (0.26%), while the gap widened in the period from 2007-2012 (0.62%). The difference is similar for all maturities.

The three-month Swap OIS is, on average, 2.56% over the period 2002-2006, while the LIBOR is 2.68%. Therefore, in this first period, LIBOR and Swap OIS are very similar, with a difference of only 12 basis points. In the period from 2007-2012, the difference increases to 43 basis points, which makes the Swap OIS rate a much better proxy for the risk-free rate in that period.

However, the three-month treasury rate is still 19 basis points below the Swap OIS in the period from 2007-2012. So why not use the treasury rate as the risk-free rate? As Hull and White [2013] argued, the treasury rate is sometimes too low with regard to the true risk-free rate, as treasuries have some special tax treatments. They estimated the credit risk on the Swap OIS rate to be around five basis points[14], which is very small. Another problem is that government bills in other countries are not as equally risk-free as they are in the US. Therefore, the Swap OIS is currently considered to be the best risk-free rate proxy that is globally comparable.

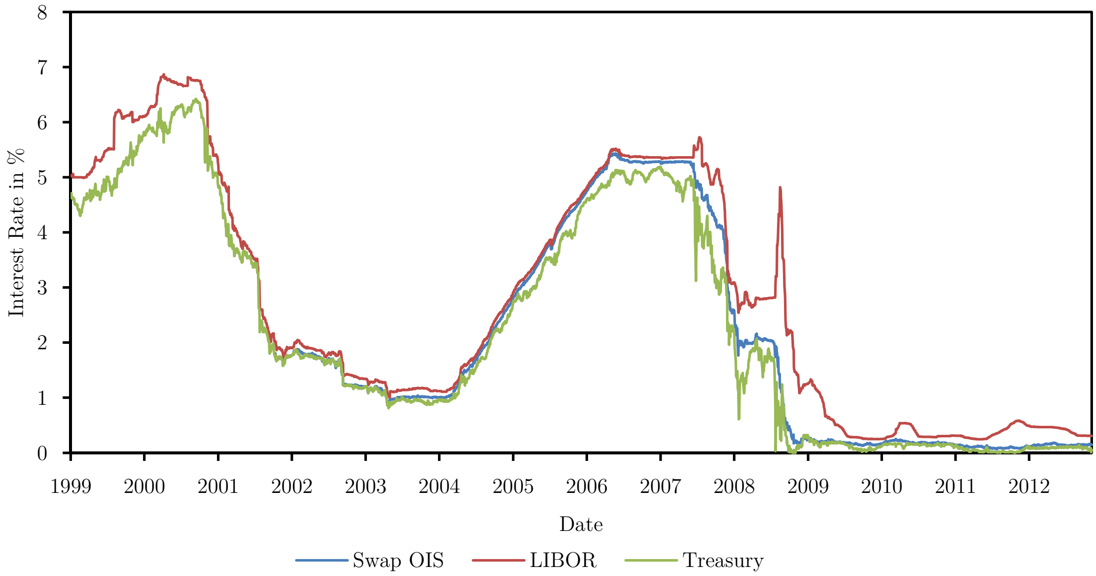
Table 7: Average US Interest Rate Levels

		6M	3M	1M	1D
1999-2012	LIBOR	2.90	2.77	2.65	N/A
	Swap	N/A	N/A	N/A	N/A
	Treasury	2.44	2.31	N/A	N/A
2002-2006	LIBOR	2.79	2.68	2.59	2.52
	Swap	2.66	2.56	2.50	2.46
	Treasury	2.58	2.42	2.33	N/A
2007-2012	LIBOR	1.86	1.67	1.50	1.38
	Swap	1.24	1.24	1.25	1.26
	Treasury	1.16	1.05	1.00	N/A

This table shows the average US interest rate over the period 1999-2012 and for the sub-periods 2002-2006 and 2007-2012. Four different maturities are presented; 6 months, 3 months, 1 month and 1 day. There are three different interest rate types: LIBOR, which is the unsecured, interbank-offered rate, the swap rate, which is an overnight index swap (OIS) on an overnight index rate, and the Treasury rate, which is the effective federal funds rate in the US. Treasury rates are the yield to maturity of US treasury bills.

[14] They estimated the credit risk by comparing the unsecured overnight rate with the secured overnight repo rate.

Figure 1: Development of the US 3-Month Interest Rates for Different Types of Instruments



This figure shows the development in the US of the LIBOR, the Swap OIS rate and the treasury rate over the period 1999-2012, with a constant maturity of three months.

III.2.7 Theoretical Futures Return

The return of a futures investment consists of the return of the futures and the collateral return, since the futures investment needs only a fraction of that of a cash investment. In the case of international investments, there is also a currency return. In these cases, the collateral can be either in the foreign currency or in the domestic currency, and the currency exposure can be constructed synthetically with a currency futures or a currency forward. If the currency risk is objectionable, the latter transactions can be omitted. The futures alone do not have a currency risk, but the profit or loss has a currency risk if it is not immediately converted into the domestic currency.

For the futures investment, a fraction of the futures notional has to be deposited into the bank as initial margin, for the protection of the bank from large, daily movements of the future. The remainder can be invested in cash on deposit. To match the maturity of the future, it is reasonable to invest the collateral with the same expiry date as the future. The precise one-day return (from t to t+1) of a futures investment that is equal to a cash total return investment in the home country is:

$$\widetilde{R} = \frac{(F_{t+1} - F_t)/exp\left(r_t \dfrac{(T-t-1)}{365}\right)}{I_t} + exp\left(r_t \frac{1}{365}\right) - 1 \tag{2}$$

$(\mathrm{F}_{t+1} - \mathrm{F}_t)/\exp\left(r_t \frac{(T-t-1)}{365}\right)$ is the present value at t+1 of the futures return, and $\exp(r_t \frac{1}{365}) - 1$ is the return for the collateral. The initial investment is the current index level (compare to Figlewski [1984], or to Hill and Naviwala [1999]).

Formula (2) has exactly the same return as a cash investment:

$$\frac{I_{t+1} - I_t + D_{t+1}}{I_t}$$

Proof: If we replace the futures price in formula (2) with formula (1), we get

$$\widetilde{R} = exp\left(r_t \frac{1}{365}\right) - 1 + \frac{\left(I_{t+1} exp\left(r_{t+1} \frac{(T-t-1)}{365}\right) - \sum_{i=t+2}^{T} D_i exp\left(r_{t+1} \frac{(T-t-2)}{365}\right) - I_t exp\left(r_t \frac{(T-t)}{365}\right) + \sum_{i=t+1}^{T} D_i exp\left(r_t \frac{(T-t-1)}{365}\right)\right)}{I_t exp\left(r_t \frac{(T-t-1)}{365}\right)}$$

as r_t is assumed to be constant over the lifetime of the futures contract, the expression can be simplified by replacing r_{t+1} with r_t:

$$= exp\left(r_t \frac{1}{365}\right) - 1 + \frac{\left(I_{t+1} exp\left(r_t \frac{(T-t-1)}{365}\right) - \sum_{i=t+2}^{T} D_i exp\left(r_t \frac{(T-t-2)}{365}\right) - I_t exp\left(r_t \frac{(T-t)}{365}\right) + \sum_{i=t+1}^{T} D_i exp\left(r_t \frac{(T-t-1)}{365}\right)\right)}{I_t exp\left(r_t \frac{(T-t-1)}{365}\right)}$$

$$= exp\left(r_t \frac{1}{365}\right) - 1 + \frac{D_{t+1} exp\left(r_t \frac{(T-t-1)}{365}\right)}{I_t exp\left(r_t \frac{(T-t-1)}{365}\right)} + \frac{\left(I_{t+1} exp\left(r_t \frac{(T-t-1)}{365}\right) - I_t exp\left(r_t \frac{(T-t)}{365}\right)\right)}{I_t exp\left(r_t \frac{(T-t-1)}{365}\right)}$$

$$= exp\left(r_t \frac{1}{365}\right) - 1 + \frac{D_{t+1}}{I_t} + * \frac{\left(I_{t+1} exp\left(r_t \frac{(T-t-1)}{365}\right) - I_t exp\left(r_t \frac{(T-t)}{365}\right)\right)}{I_t * exp\left(r_t \frac{(T-t-1)}{365}\right)}$$

$$= exp\left(r_t \frac{1}{365}\right) - 1 + \frac{D_{t+1}}{I_t} + \frac{I_{t+1}}{I_t} - exp\left(r_t \frac{1}{365}\right)$$

$$= \frac{D_{t+1}}{I_t} + \frac{(I_{t+1})}{I_t} - 1$$

which is exactly the same as the return of a cash investment.

Similarly, I_t can be replaced in (2) with

$$I_t = \left(F_t + \sum_{i=t+1}^{T} D_i exp\left(r_t \frac{(T-t)}{365} \right) \right) / exp\left(r_t \frac{(T-t)}{365} \right)$$

from (1). We then get:

$$\widetilde{R} = \frac{(F_{t+1} - F_t)/exp\left(r_t \frac{(T-t-1)}{365} \right) * exp\left(r_t \frac{T-t}{365} \right)}{F_t + \sum_{i=t+1}^{T} D_i exp\left(r_t \frac{(T-t)}{365} \right)} + exp\left(r_t \frac{1}{365} \right) - 1$$

$$= \frac{(F_{t+1} - F_t) * exp\left(r_t \frac{1}{365} \right)}{F_t + \sum_{i=t+1}^{T} D_i exp\left(r_t \frac{(T-t-1)}{365} \right)} + exp\left(r_t \frac{1}{365} \right) - 1$$

$$\widetilde{R} = \left(1 + \frac{(F_{t+1} - F_t)}{F_t + \sum_{i=t+1}^{T} D_i exp\left(r_t \frac{(T-t-1)}{365} \right)} \right) * exp\left(r_t \frac{1}{365} \right) - 1 \qquad (3)$$

Theoretically, (2) and (3) are identical. In practice, (3) is superior to (2), since mispriced futures are better integrated. Suppose, for example, that the futures price is 1000 and the index price is 1000. There are no dividends and no interest. If the futures price increases to 1100 as a result of mispricing, that is a gain of 100, which is 10% in both (2) and (3). However, if the futures price falls back to 1000, that is a loss of 100, which is -10% in (2) and -9.09% in (3). Thus, it would be a geometrical return of -1% in (2) and 0% in (3). Obviously, there should not be any loss, since a gain of 100 plus a loss of 100 is 0.

How much should I invest if I want to replicate a cash investment with a futures? The investment amount has to be derived from the index. Let us assume that the index level is 1000, the futures price is 1050, and the contract size[15] of the futures is 10. In this case, one futures has to be bought to replicate a $ 10'000 cash investment. After every dividend payment, the futures investment has to be increased to replicate the reinvestment of the dividends in a total return index.

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[15] The contract size of the futures specifies the relation to the index. If the contract size is 10, then one futures is worth 10 times the index. Contract sizes of the equity futures in this analysis vary between 10 and 10'000.

III.2.8 Dividend Uncertainty

The determination of the futures price requires knowledge regarding the futures dividend payment. However, such dividends are uncertain. In the theory of external habit models, volatile dividends are an important factor for the explanation of the equity premium puzzle. The external habit model is able to explain the high equity premium under certain assumptions with the relationship between consumption claims and dividend earnings. The problem is that the model needs to make certain assumptions for parameters to solve for the price-dividend ratio. With futures prices, this can be measured directly with the implied dividend in the future. Suppose that there exists a stochastic discount factor of the form

$$B_t(C_{t+1}/C_t)^{-G_t}$$

for some time-varying but non-random B_t and $G_t > 0$, as in the external habit model by Campbell and Cochrane [1999]. G_t is the risk-aversion factor. Then, the price P_t of an asset that pays a single risky dividend is (compare this to Wachter [2005])

$$P_t = E_t\left(B_t\left(\frac{C_{t+1}}{C_t}\right)^{-G_t} D_{t+1}\right)$$

while the price of a one-period risk-free bond is

$$P_t^f = E_t\left(B_t\left(\frac{C_{t+1}}{C_t}\right)^{-G_t} D_{t+1}\right)$$

Suppose that log consumption growth is a random walk, with normally distributed changes. Log-dividend growth is a normally-distributed deviation, time-varying expected value μ_t. The joint specification is

$$logC_{t+1} = \mu_C + logC_t + \varepsilon_C$$

$$logD_{t+1} = \mu_t + \varepsilon_D$$

Let σ_C^2 and σ_D^2 be the variance of ε_C and ε_D respectively, and let σ_{CD} be the covariance.

The products of the lognormal random variables are lognormal, as is the power, so the insides of both expectations are lognormal. Recall that for a lognormal random variable X, if log X has mean μ and variance σ^2, then

$$logE(X) = \mu + \sigma^2/2$$

By this result,

$$logP_t = logB_t + \mu_t - G_t\mu_C + (\sigma_D^2 - 2G_t\sigma_{CD} + G_t^2\sigma_C^2)/2 \tag{4}$$

and

$$logP_t^f = logB_t - G_t\mu_C + G_t^2\sigma_C^2/2 \tag{5}$$

If we consider the log ratio of (3) and (4), this can be simplified to

$$log\frac{P_t}{P_t^f} = \mu_t + (\sigma_D^2 - 2G_t\sigma_{CD})/2 \tag{6}$$

Thus the market price of $E_t(\mathrm{D_t})$ is equal to the dividend yield μ_t minus the risk premium $(\sigma_D^2 - 2G_t\sigma_{CD})/2$. We can solve equation (5) for G_t:

$$G_t = \left(\frac{\sigma_D^2}{2} - \left(log\frac{P_t}{P_t^f} - \mu_t\right)\right)/\sigma_{CD} \tag{7}$$

Thus, futures have the unique property of implicitly measuring the market price of the future dividends. While Campbell and Cochrane [1999] and Wachter [2005] needed to make assumptions for the parameters in order to estimate G_t, with futures data G_t can be observed within this data in chapter 3.3.

III.3 Efficiency of Synthetic Investments

III.3.1 Price

The difference of the theoretical futures price according to the cost of carry model and the actual market futures price shows whether a futures is fairly priced or not. Table 8 presents the standard error of this day-end difference in the period from July 2009 to June 2010. A higher standard error indicates that the futures price deviated to a greater degree from its fair value. For the interest rate, I used the daily three-month interest rate for three--month contracts and the one-month rate for one- month contracts, as the term premium is very small at the short end of the curve (compare this to Table 7).

The historical prices show that the standard error of the difference between the theoretical and the actual futures price is small. For 19 of 22 futures, it is lower than the round-trip transaction costs, which means the standard error lies within the theoretical no-arbitrage bounds. For monthly expiring futures (in the big markets like Spain and France), the standard error is generally smaller. Higher transaction costs also lead to higher volatility, since the deviation must be larger in order for arbitrageurs to make a profitable arbitrage trade. Greece has the highest volatility, as well as the highest transaction costs. Thus, there was rarely an arbitrage opportunity during this period.

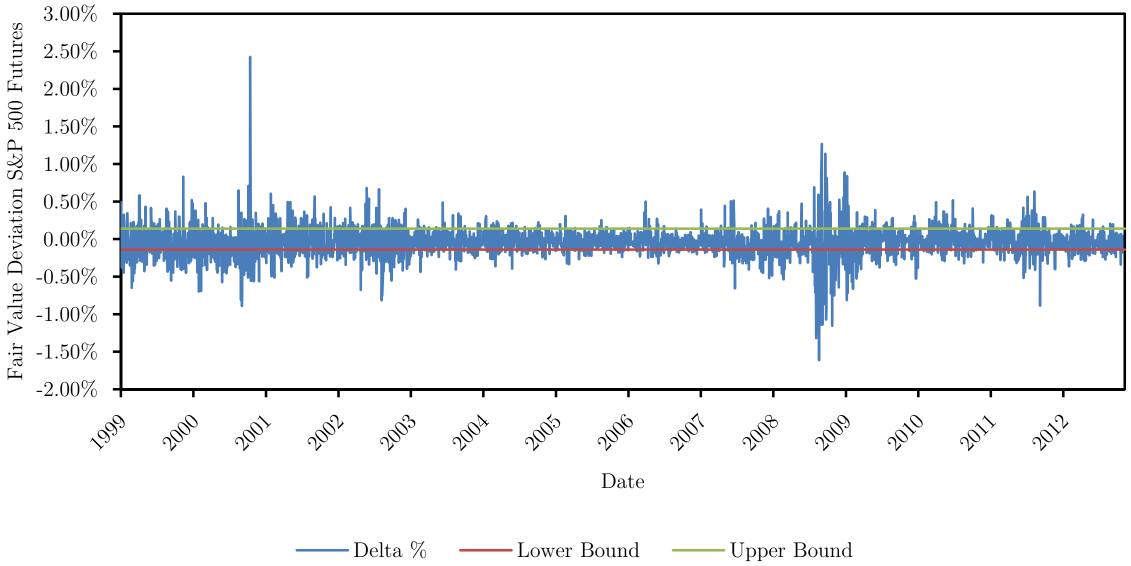
Table 8: Fair Value Deviation from Futures Price

Country	Standard Error	Round Trip Transaction Costs
USA	0.14%	0.14%
Japan	0.26%	0.29%
UK	0.14%	0.68%
France	0.05%	0.15%
Canada	0.20%	0.15%
Australia	0.28%	0.20%
Germany	0.14%	0.13%
Switzerland	0.14%	0.18%
Spain	0.09%	0.17%
Italy	0.12%	0.19%
Netherlands	0.07%	0.23%
Sweden	0.16%	0.29%
Hong Kong	0.29%	0.43%
Singapore	0.31%	0.42%
Belgium	0.33%	0.45%
Denmark	0.23%	0.81%
Norway	0.52%	0.48%
Greece	0.57%	0.82%
Austria	0.30%	0.71%
Portugal	0.22%	0.49%
Finland	0.14%	0.54%
Eurozone	0.14%	0.19%

The second column in the table shows the standard error of the difference between the theoretical and the actual futures price, measured during the period from July 2009 to June 2010. The third column presents the round-trip transaction costs for an arbitrage trade, which incorporates the buying and selling of a future, and the buying and selling of cash stocks, according to the transaction costs in Table 4.

Figure 2 shows the daily fair value deviation for the S&P 500 Futures from 1999-2012. The deviation during the financial crisis of 2007-2008 was much higher than it was before. One problem is that the futures index in the USA closes 15 minutes later than does the cash index. The volatility increased during the crises, which increased the deviation in those 15 minutes. However, Figure 3 shows that, most of the time, the deviation between the S&P 500 Futures and the Index at 4 pm is similar to the closing price deviation. The standard error of the deviation in 2008 is 0.36% at the 4 pm futures price and 0.37% at the 4:15 pm futures price.

Figure 2: Fair Value Deviation S&P 500 Futures



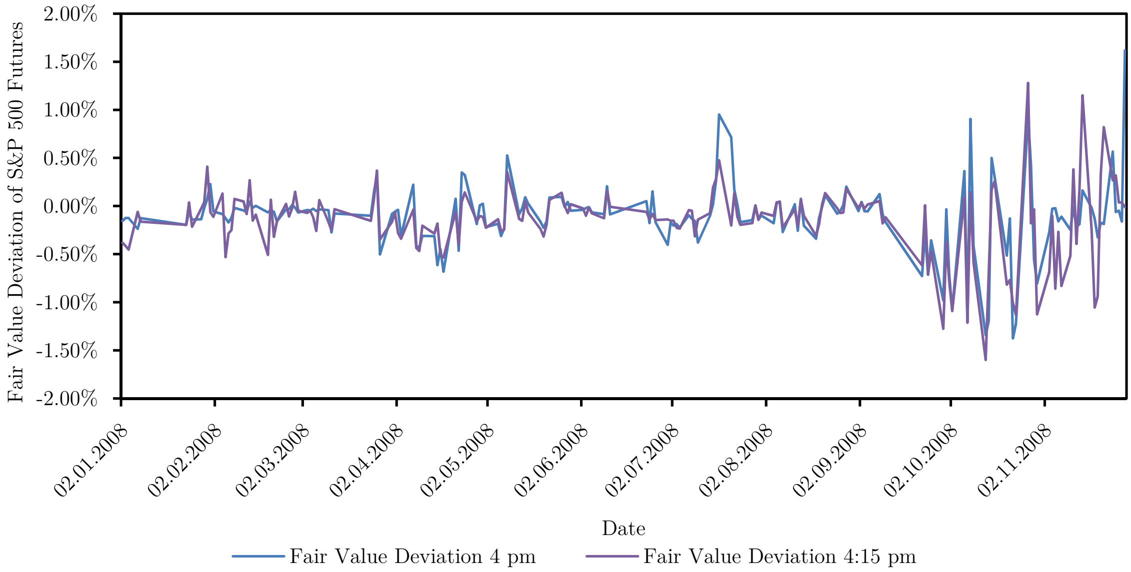
This figure shows the basis of the S&P 500 Futures over the period from March 1999 to December 2012. The basis is the difference between the theoretical futures price and the actual market futures price. The lower and upper bound is the range wherein arbitrage is not directly possible due to transaction costs.

Furthermore, transaction costs were higher during the crisis, due to higher bid-ask spreads. Nonetheless, there might have been some arbitrage opportunities, but arbitrageurs were probably not capable of conducting business given the lack of trading limits. Over the entire sample period from February 26 1999 to December 31 2012, the standard deviation was 0.21%, which is similar to the 0.20% in the study by Hill and Naviwala [1999].

Figure 4 shows the fair value deviation for a smaller market, the Austrian ATX index. While the deviation has usually been within the no-arbitrage bounds, there was an astonishing spike during the financial crisis in the period from September to November 2008, where the fair value of the futures rose permanently, up to 10% above the actual futures price. As there are no dividend payouts in the last quarter for the ATX, it cannot be the estimation uncertainty that caused this deviation. However, the ATX Index had lost more than 50% in the month following the Lehman collapse. The prohibition of short sales might have had some influence.

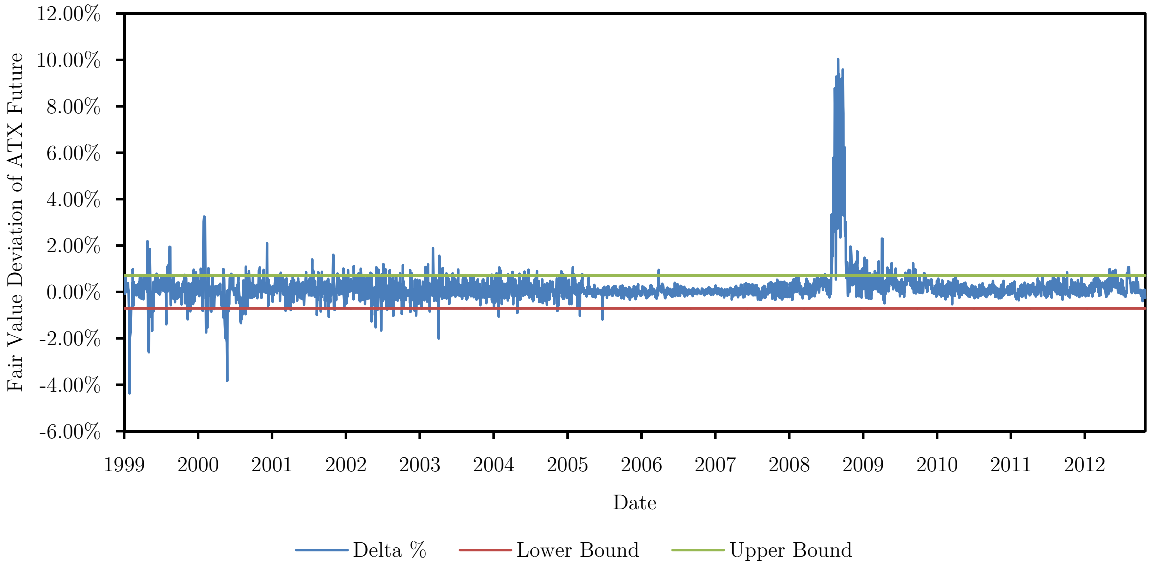
As we have seen so far, the arbitrage opportunities in large markets like the S&P 500 have been very limited; thus, the price efficiency is quite high. For smaller markets like the ATX, here was a huge opportunity in 2008, at least for investors who wanted to buy, as there was a short-sales prohibition.

Figure 3: End of Day Deviation in 2008



This figure shows the fair value deviation of the S&P 500 Futures at 4 pm and 4:15 pm for the year 2008. The fair value deviation is the difference between the theoretical futures price and the actual market futures price. The S&P 500 Index has its closing price at 4 pm and the S&P 500 Futures has its closing price at 4:15 pm. Therefore, the price of the S&P 500 Futures at 4 pm is an intraday price.

Figure 4: Basis ATX Futures 1999-2010



This figure shows the fair value deviation of the ATX Futures over the period from March 1999 to December 2012. The fair value deviation is the difference between the theoretical futures price and the actual market futures price. The lower and upper bound is the range wherein arbitrage is not directly possible due to transaction costs.

III.3.2 Return

Table 9 presents the historical returns of the returns futures, their indices, and the related MSCI country index, using formula (2) to calculate the futures returns. This allows us to see exactly whether the futures return is above or below the total return index. I used monthly data and annualized them by multiplying the arithmetic average by 12.

Table 9: Historical Futures and Index Returns 1999-2012

Country	Index TR	Index NR	Futures TR	MSCI TR
USA	4.13%	3.58%	3.81%	3.97%
Japan	3.43%	3.30%	3.29%	3.13%
UK	4.65%	4.62%	4.39%	4.73%
France	6.28%	5.59%	6.19%	6.56%
Canada	11.88%	11.35%	11.78%	12.20%
Australia	15.06%	14.19%	14.66%	15.16%
Germany	7.97%	7.35%	7.94%	7.93%
Switzerland	6.79%	6.12%	6.35%	6.89%
Spain	6.93%	6.19%	6.94%	7.59%
Italy	2.81%	1.63%	2.81%	3.25%
Netherlands	4.33%	3.69%	4.04%	5.77%
Sweden	12.21%	11.14%	12.48%	12.16%
Hong Kong	12.14%	12.14%	12.15%	11.71%
Singapore	12.59%	12.59%	12.06%	13.90%
Belgium	5.29%	4.45%	4.91%	4.54%
Denmark	12.09%	11.59%	12.05%	12.70%
Norway	16.81%	16.03%	17.46%	16.02%
Greece	-11.96%	-12.45%	-8.65%	-19.06%
Austria	12.73%	12.11%	13.76%	8.53%
Portugal	2.43%	1.66%	2.03%	2.42%
Finland	6.25%	5.04%	9.35%	3.54%
Eurozone	4.56%	4.04%	4.29%	5.85%

This table presents the annualized return in USD from 1999-2012. Index TR is the total return index, meaning all dividends are fully reinvested. Index NR is the net return index, meaning only the net dividend amount of withholding taxes is reinvested. The Futures TR is the total return of a futures investment. MSCI TR is the total return index of the MSCI country index corresponding to the country of origin of the futures.

In most countries, the futures return is below the index gross return, with the exceptions of Italy, Spain, Sweden, Hong Kong, Norway, Austria, and Finland. However, the futures return is above the index net return in most countries, except for Japan, the UK, Singapore, and Greece. The UK and Singapore are special cases, since the gross and net returns are essentially equal, as they are in Hong Kong, because the withholding tax rate is 0.

Table 10: Tracking Error of Futures, Indices, and MSCI

Country	Futures vs. Index	Index vs. MSCI	Futures vs. MSCI
USA	0.37%	0.74%	0.82%
Japan	1.72%	2.67%	3.46%
UK	1.11%	1.12%	1.72%
France	0.92%	1.51%	1.89%
Canada	0.95%	1.55%	1.74%
Australia	1.57%	1.57%	2.45%
Germany	1.36%	1.73%	2.14%
Switzerland	1.34%	1.14%	1.90%
Spain	1.09%	2.74%	3.11%
Italy	0.69%	1.73%	1.98%
Netherlands	0.73%	4.49%	4.57%
Sweden	1.47%	2.23%	2.72%
Hong Kong	1.88%	8.22%	8.46%
Singapore	2.48%	3.52%	4.24%
Belgium	1.39%	5.48%	5.71%
Denmark	3.28%	4.63%	5.39%
Norway	3.13%	3.44%	5.01%
Greece	4.24%	17.94%	18.34%
Austria	4.34%	4.46%	6.69%
Portugal	1.69%	4.10%	4.34%
Finland	3.02%	10.75%	10.49%
Eurozone	1.43%	6.03%	6.16%

This table presents the annualized tracking error of futures, indices and the MSCI. The period is February 1999 to December 2012. The tracking error is the volatility of the return difference between two return series.

Table 10 presents the historical tracking error of the returns futures, their indices, and the related MSCI country index. The tracking error between the futures and the index arises either because of futures mispricing, or as a result of different closing times between the futures and the index. This tracking error is random. The tracking error between the S&P 500 Futures and the S&P 500 index is 0.37% and is thus the smallest among all countries[16]. The highest tracking error is that of Austria with 4.34%, which is also one of the countries with the highest transaction costs and fair value deviation (remember Table 8).

[16] It is standard in the financial industry to measure the tracking error using monthly data. However, I acknowledge the objection of Hill and Naviwala [1999], in that the monthly measurement overstates the tracking error of futures due to the mean reversion of price deviations. Using quarterly measurements, the tracking error would actually decline to 0.23% for the S&P 500 Futures or 0.87% for the TOPIX Futures.

The tracking error between the index and the MSCI shows how well the constituents in the index cover the ones in the MSCI country index. Remember from Table 5 that the USA, the UK, Spain, Italy, the Netherlands, Sweden, Hong Kong, and Singapore were countries in which the number of shares in the index was close[17] to that in the MSCI? Nevertheless, Hong Kong and the Netherlands have a high tracking error between the index and the MSCI. In the Netherlands, the AEX Index is not market-capitalization weighted but price-weighted, producing the high tracking error of 4.49%. In Hong Kong, the Hang Seng Index includes all Hong Kong listed stocks with primary listings, while MSCI Kong excludes Hang Seng-listed Chinese stocks, such as H-shares (companies incorporated into Mainland China) and red-chips (incorporated overseas but controlled by the Chinese government).

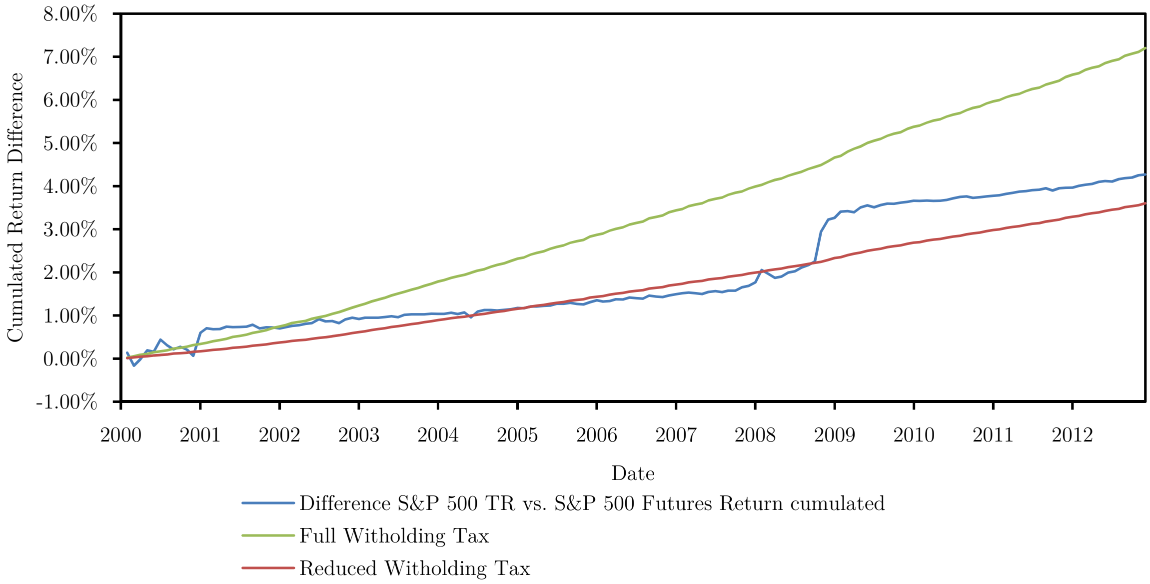
The tracking error between the futures and the MSCI index is the combination of the factors mentioned above. Because the factors are not perfectly correlated but are independent, this tracking error is smaller than is the sum of the individual tracking errors. The tracking error problem arises due to mispricing of the futures, different closing times, and different constituents between the futures and the MSCI. As Table 10 shows, the constituent mismatch is the main driver of the tracking error, since the deviation between the Index and the MSCI is larger than is the deviation between the futures and the Index.

Figure 5 shows the historical, cumulated withholding taxes and the return differences between the S&P 500 gross return index and the S&P 500 Futures return, computed with formula (2). The data are from January 2000 to December 2012. The cumulated full withholding tax is the difference between the MSCI gross and net returns. The reduced withholding tax is 50% of the full withholding tax, accounting for the double tax agreements.

There are two sources of return differences between the futures and the gross return index. The first and most important is the mispricing of the futures on the rollover day. This leads to a permanent return difference. The second is the mispricing of the futures on the measurement day, which here is the end of month. The second error is only temporary, given that there is no mispricing on the rollover day. The mispricing of the futures may be for several reasons: firstly, the dividends are uncertain on the rollover day, therefore the dividend estimations could be different from the realized dividends. Furthermore, some market participants receive net dividends, while others receive gross dividends. Thus, the fair value of futures is not equal for all participants. The interest rates might also be different for different participants. In addition, possibly the most important reason is a different offer-demand structure in the futures market than in the cash one. Arbitrageurs can only offset this difference up to the arbitrage bounds.

[17] Less than 20% deviation in the number of companies.

Figure 5: Cumulated Difference between S&P 500 Gross and Futures Returns



This figure compares the withholding tax impact. The green line is the cumulated full withholding of the S&P 500. The red line is the cumulated, reduced withholding tax, which is half of the full withholding tax. The blue line is the return difference between the S&P 500 total return index and the futures return. The period is January 2000 to December 2012.

The return of the S&P 500 Futures is significantly lower than is the S&P 500 total gross return index, at a 95% significance level, but is also significantly higher than the net return. However, it is not significantly different from the middle path between the gross and net return; in other words, the return with the reduced withholding tax rate. Thus, for investors who benefit from the reduced withholding tax rate – as most investors do – the S&P 500 Futures achieves an equal return to that which they would receive when investing in cash equities. For investors who pay the full withholding tax rate, the futures investment achieves a superior return.

The increase of the difference in autumn 2008[18] was caused by the mispricing of the rollover spread. The rollover of the futures contract from September to December 2008 was on September 18. On this day, the December contract was six Index points above the September contract, while the fair value of the December contract should have been 1 point below the September contract. These six points are a loss of 0.5%. Therefore, the key point is that the implied dividend from the December contract was 0.5% too low, causing the price of the contract to be 0.5% too high.

[18] 24 bps loss in October and 7 bps in November, compared to the reduced withholding tax loss.

Interestingly, the overpricing would have disappeared if the LIBOR had been used instead of the Swap rate, as it was 3.2%, compared to the swap rate at 1.6%.[19] If market participants used the LIBOR to calculate the fair value of the futures price, this could have explained the difference.[20] The overpricing was persistent during the days before rollover, which confirms this observation. As the futures market was 38 bps overpriced on September 30, the loss did not occur in September but in October and November, since the futures were still 20 bps overpriced on October 31. The index lost 17% in October and 7.5% in November, and the volatility was 76% during that period.

Figure 6 shows the same data for the EURO STOXX 50 and Figure for the FTSE 100, an index with no withholding taxes. The return of the EURO STOXX 50 Futures and the gross return index is not significantly different, neither is it different from the reduced withholding tax return. However, the return difference between the index total return and the futures return again resembles the reduced withholding tax. Again, in 2008, there was a lower return in the EURO STOXX 50 Futures than there was in the cash index investment.

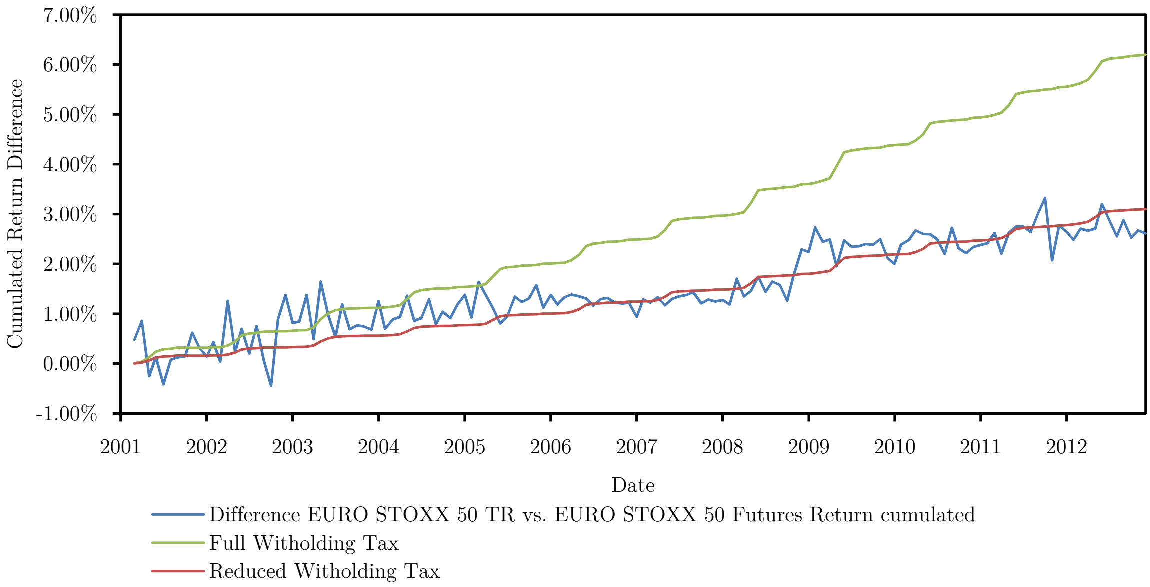
For the FTSE Futures, the September futures were 5.9 points above the fair value during the rollover of the futures from September to December on 18 September 2008, while the December futures were 28.7 above the fair value. This difference equates to 0.46%. However, as the FTSE 100 index has no withholding taxes, the tax difference cannot be the explanation for the worse return of the futures compared to the cash investment.

Figure 8 shows the same data for the DAX, a total return index. The DAX Futures is the only futures that does not suffer from dividend uncertainty, as the futures refers to a total return index. As can be seen in Figure 8, the return of the DAX Futures tracks the return of its corresponding DAX index very closely. Thus, the main reason for the return difference in futures and cash investments is not that some participants might have priced the futures according to LIBOR instead of using true risk-free interest rates. Otherwise, we should see similar differences in the DAX Futures and in the EURO STOXX 50 Futures. However, the explanation is to be found in the dividend uncertainty. Investors have been more risk-averse or have underestimated the dividends during the financial crisis, causing futures prices to be too high and therefore their returns to be too low. As the DAX Futures have no dividend uncertainty, its futures return racks the index return more closely, and there was no major difference during the financial crisis.

[19] This was in the middle of the financial crisis, when the LIBOR rate disconnected sharply from the risk-free rate.

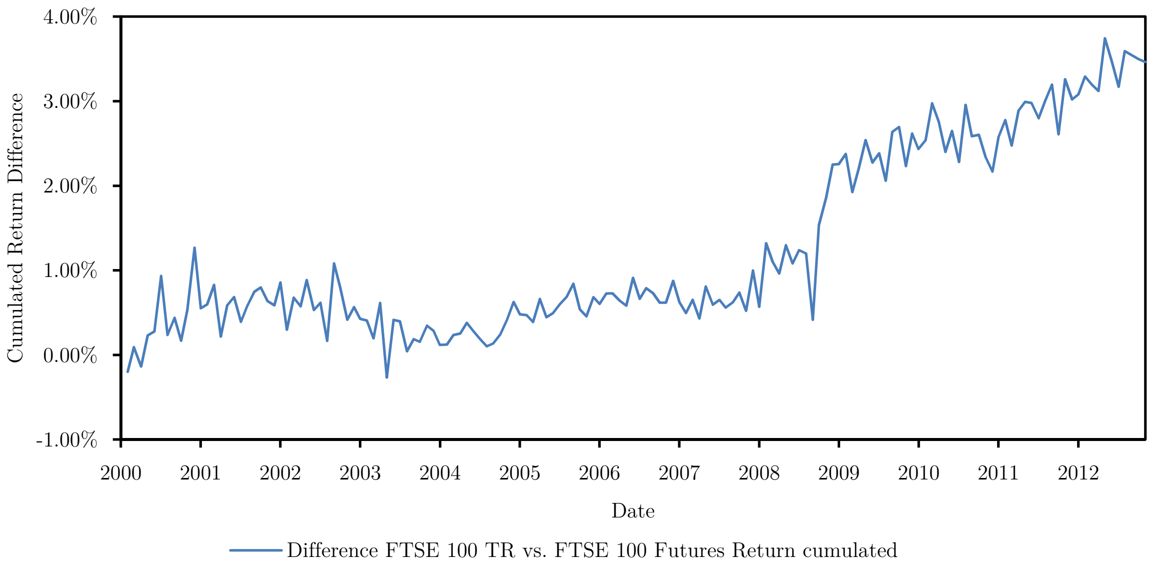
[20] Obviously, investors can invest in the LIBOR instead of a pure risk-free rate, but this investment is more risky than is the corresponding cash investment, since there is additional counterparty risk from the money invested in the LIBOR market.

Figure 6: Cumulated Difference between EURO STOXX 50 Gross and Futures Returns



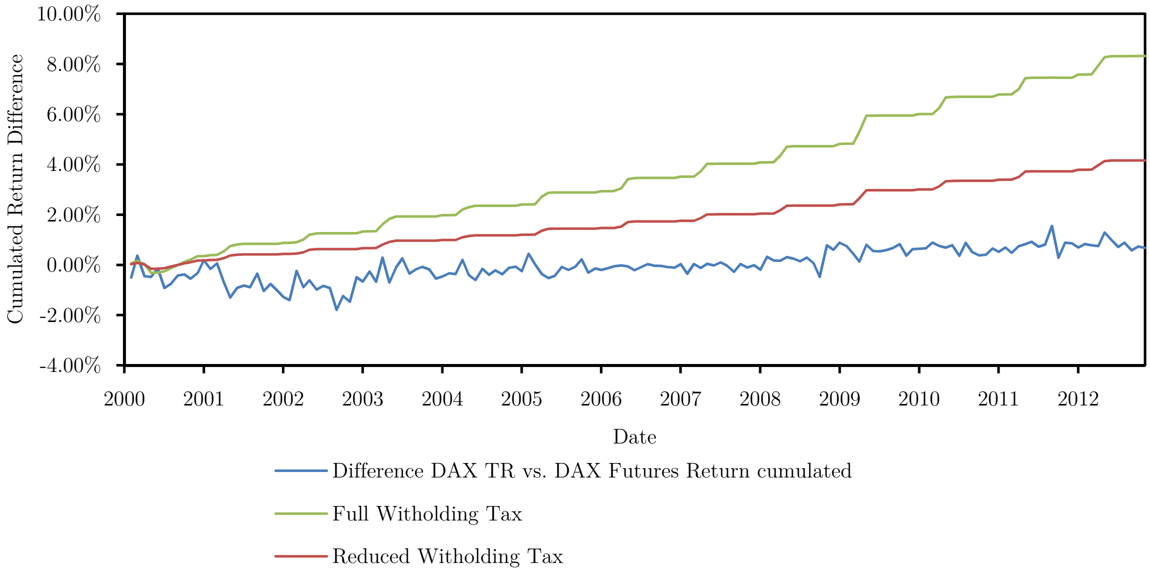
This figure compares the withholding tax impact. The green line is the cumulated full withholding of the EURO STOXX 50. The red line is the cumulated reduced withholding tax, which is half of the full withholding tax. The blue line is the return difference between the EURO STOXX 50 total return index and the futures return. The period is April 2001 to December 2012.

Figure 7: Cumulated Difference between FTSE 100 Gross and Futures Returns



This figure compares the withholding tax impact. The green line is the cumulated full withholding of the FTSE 100. The red line is the cumulated reduced withholding tax, which is half of the full withholding tax. The blue line is the return difference between the FTSE 100 total return index and the futures return.

Figure 8: Cumulated Difference between DAX Gross and Futures Returns



This figure compares the withholding tax impact. The green line is the cumulated full withholding of the DAX. The red line is the cumulated reduced withholding tax, which is half of the full withholding tax. The blue line is the return difference between the DAX total return index and the futures return.

III.3.3 Dividends

A dividend analysis is appropriate, as we have seen in chapter III.3.2 that dividends are the main reason for return differences between futures and cash investments. As all parameters except dividends are known, the futures implied dividend can be measured with the futures price, the interest rate, and the index value.

Remember from (1) that the futures price is

$$F_t^* = I_t exp\left(r_t \frac{(T-t)}{365}\right) - \sum_{i=t+1}^{T} D_i exp\left(r_t \frac{(T-t-1)}{365}\right)$$

Thus, the market estimation of dividends implied by the futures price from t+1 to T is

$$\sum_{i=t+1}^{T} D_i exp\left(r_t \frac{(T-t-1)}{365}\right) = I_t exp\left(r_t \frac{(T-t)}{365}\right) - F_t$$

The problem is that the futures price is too volatile for an exact measurement of D, as the offer-demand structure in the futures market can be different from that of the cash one, and the day-end value is measured at a slightly different point in time (compare this to chapter 2.6). This problem can be solved by looking at the difference between the implied dividend of both the first and the second futures contracts. This difference is much more stable. The implied dividend for futures 1 ($F_{t,1}$) and for futures 2 ($F_{t,2}$) is

$$\sum_{i=t+1}^{T1} D_i exp\left(r_t \frac{(T1-t-1)}{365}\right) = I_t exp\left(r_t \frac{(T1-t)}{365}\right) - F_{t,1}$$

$$\sum_{i=t+1}^{T2} D_i exp\left(r_t \frac{(T2-t-1)}{365}\right) = I_t exp\left(r_t \frac{(T2-t)}{365}\right) - F_{t,2}$$

where T1 is the expiration date of futures 1 and T2 is the expiration date of futures 2.

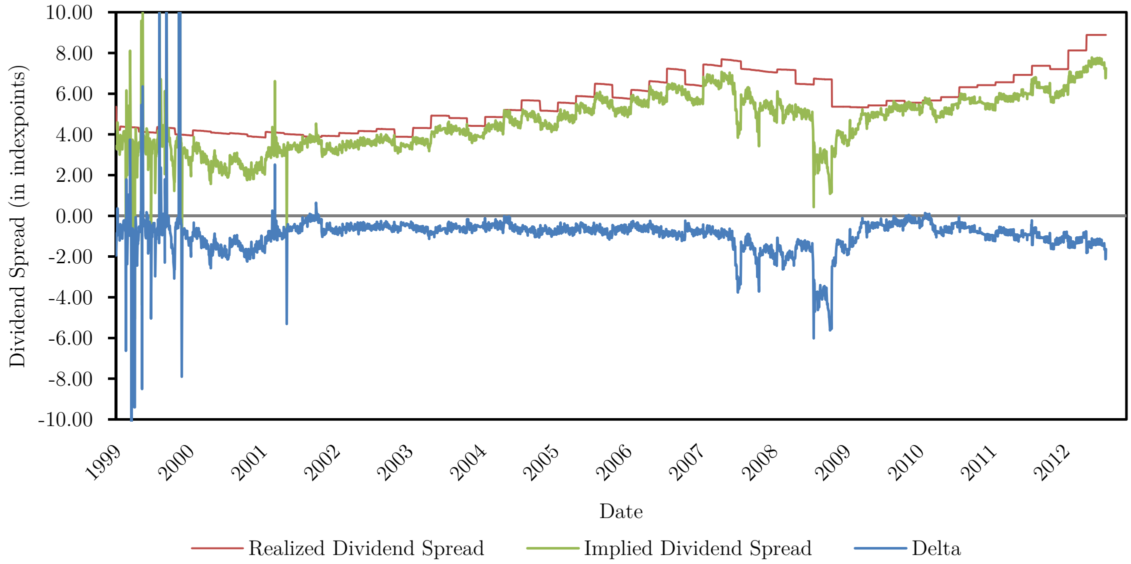
The implied dividend spread between futures 1 ($F_{t,1}$) and futures 2 ($F_{t,2}$) is determined as follows:

$$\begin{gathered}
\sum_{i=t+1}^{T2} D_i exp\left(r_t \frac{(T2-t-1)}{365}\right) - \sum_{i=t+1}^{T1} D_i exp\left(r_t \frac{(T1-t-1)}{365}\right) \\
= I_t exp\left(r_t \frac{(T2-t)}{365}\right) - F_{t,2} - I_t exp\left(r_t \frac{(T1-t)}{365}\right) + F_{t,1} \\
\Longleftrightarrow \\
\sum_{i=t+1}^{T2} D_i exp\left(r_t \frac{(T2-t-1)}{365}\right) - \sum_{i=t+1}^{T1} D_i exp\left(r_t \frac{(T1-t-1)}{365}\right) \\
= I_t \left(exp\left(r_t \frac{(T2-t)}{365}\right) - exp\left(r_t \frac{(T1-t)}{365}\right)\right) + F_{t,1} - F_{t,2} \\
\approx \\
\sum_{i=T1+1}^{T2} D_i exp\left(r_t \frac{(T2-t-1)}{365}\right) \approx I_t \left(r_t \frac{(T2-T1)}{365}\right) + F_{t,1} - F_{t,2}
\end{gathered} \tag{8}$$

Thus, the dividends from T1+1 to T2 are approximately the price difference between futures 1 and 2, plus the interest earnings over that period. The measurement error between futures and index prices is almost offset as we construct the differences between the two futures prices.

Figure 9 presents this difference for the S&P 500 Futures and compares it to the actually realized dividend amount in the period between futures 1 and futures 2. The delta between the futures implied dividend spread and the realized dividend spread shows some spikes in the period from 1999 to 2001, caused by minor data quality. The futures implied dividend spread is usually below the realized dividend, and on average, the delta is -0.56 index points or -0.04% of the index value. From 2002 to 2006, the delta was between 0 and -1 index points, which is very small as the tick size is only 0.25 index points. In 2008, the delta became much more negative at 6.24 index points, respectively 0.46% of the index value. The negative delta means that either the dividends have been underestimated, or the uncertainty regarding dividends caused this discount due to risk aversion. This explains why the futures performance was slightly worse than was the cash index performance, as the second futures is always slightly too expensive. As there is no uncertainty at expiration, this leads to a permanent loss.

Figure 9: Realized and Futures Implied Dividends for S&P 500

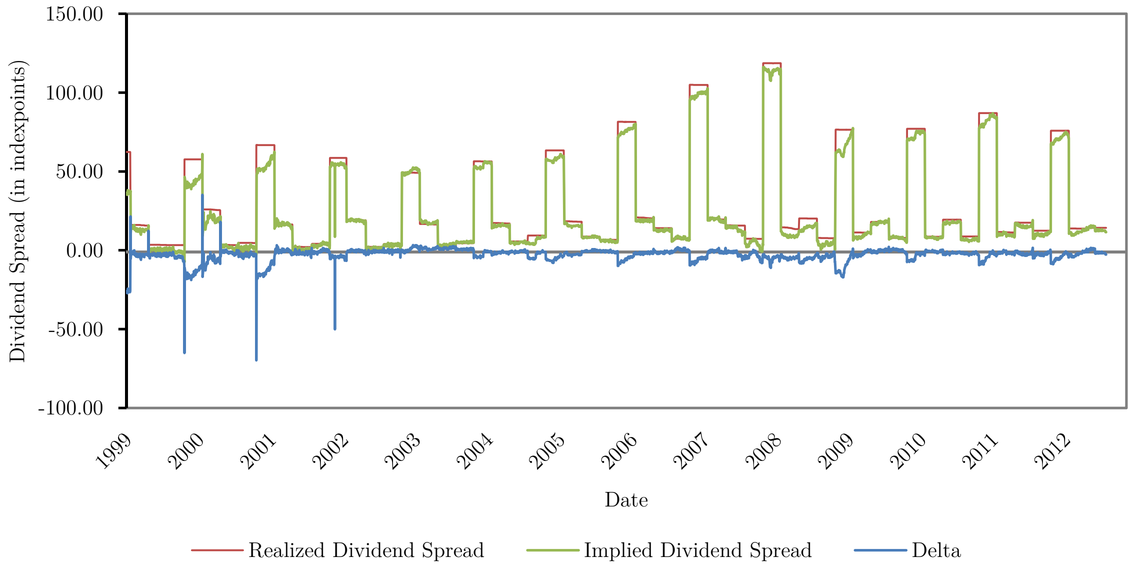


This figure compares the dividend spread between the first and second S&P 500 Futures contracts. The realized dividend spread is the future value of the compounded realized dividends from the expiry of the first futures contract until the expiry of the second futures contract. The futures implied dividends are the implicit dividends in the same period, given the market price of the index, the futures, and the interest rates. Delta is the difference between the futures implied dividend spread and the realized dividend spread. The period is from March 1999 to December 2012.

Figure 10 presents the same analysis for the EURO STOXX 50. In contrast to the S&P 500, dividends are not paid regularly, as most dividends in Europe are paid in the first quarter, while the dividend amount in the other quarters is minimal. With regard to the delta between the futures implied dividend spread and the realized dividend spread, it is also permanently negative and, on average, is -2.88 index points, or -0.08% respectively. It had peaks in 2000, 2001, and 2008. In 2008, the delta had a peak with 17.11 index points, 0.77% respectively. Thus, similarly to the S&P 500, dividends had been underestimated or risk aversion had increased.

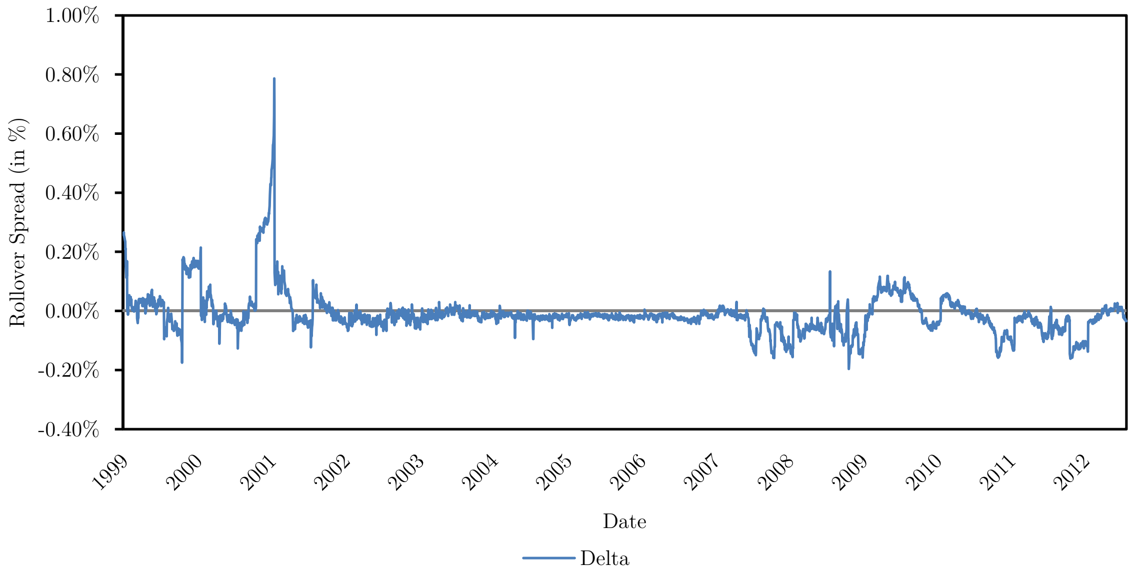
Figure 11 presents the same analysis for the DAX Futures. The delta is also computed with formula (8) but, as this index has no dividend estimation problematic, the left-hand side of the equation is zero. Thus, the realized dividend spread is always zero. Therefore, for the DAX Futures, the delta is just the pricing error between the first and second futures. The main differences compared to the EURO STOXX 50 and the S&P 500 Futures are that the delta is not permanently negative, and its magnitude is smaller. On average, the delta is -0.6 index points, or -0.01%.

Figure 10: Realized and Futures Implied Dividends for EURO STOXX 50



This figure compares the dividend spread between the first and second EURO STOXX 50 Futures contracts. The realized dividend spread is the future value of the compounded realized dividends from the expiry of the first futures contract until the expiry of the second futures contract. The futures implied dividends are the implicit dividends in the same period, given the market price of the index, the futures, and the interest rates. Delta is the difference between the futures implied dividend spread and the realized dividend spread.

Figure 11: Spread for DAX Futures



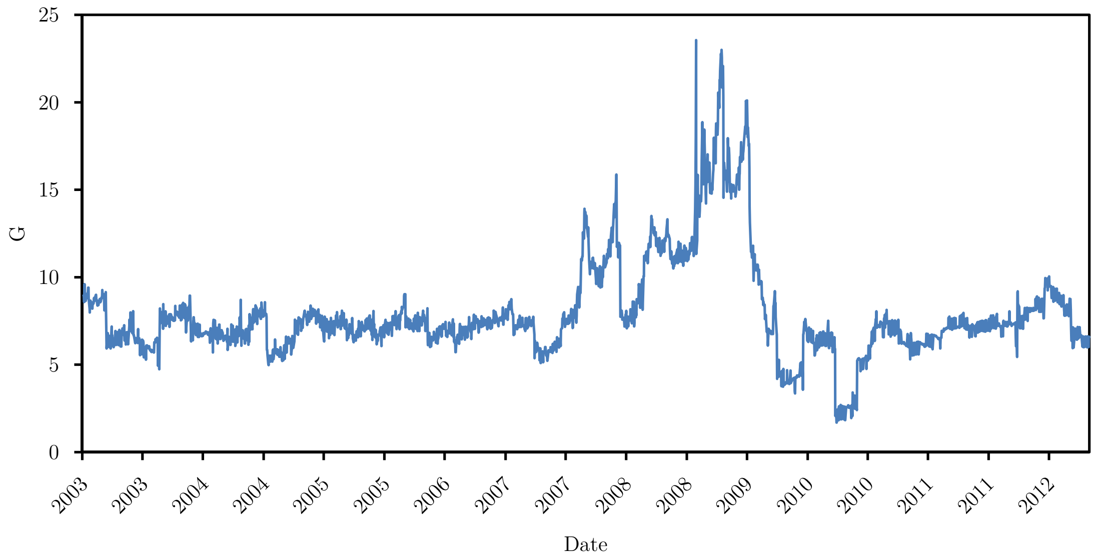
This figure compares the spread between the first and second DAX Futures contracts. As the DAX Futures has no dividends, the realized spread should be zero. The delta is the implicit spread in the period between the expiry of the first futures contract and the expiry of the second futures contract, given the market price of the index, the futures, and the interest rates.

In order to determine the effect of risk aversion on futures implied dividends, we can take a closer look at the S&P 500. The S&P 500 has a unique property in that it is extremely liquid and has continuous dividends in each quarter. This allows the determination of risk aversion in the external habit model (compare this to chapter 2.8).

I used formula (7) to compute the values for the risk aversion factor G_t. For the parameters σ_D^2, and σ_{CD}, I took the long-term dividend growth and the consumption growth data from the website of Robert Shiller[21]. This dataset is from 1890-2009, and has the following quarterly values: $\sigma_D^2 = 0.34\%$, and $\sigma_{CD} = 0.03\%$. For μ_t, I used the dividend from one year ago plus the divided growth of the previous quarter (compared to that of the year ago).

Given this, the value of G_t is calcuated for the S&P 500 and is presented in Figure 12.

Figure 12: Implied Risk Aversion of S&P 50 Dividend Spread



This figure presents the implied risk aversion of the S&P 500, derived by the dividend spread of the first and second futures contracts. G_t is the risk-aversion factor. The period is from December 2002 to December 2012.

The average value of G_t for the S&P 500 is 7.9, which is far below the estimates of Campbell and Cochrane [1999], as their estimate is around 40. Only during the financial crisis did G_t increase to a maximum 23.6. Whether the increasing risk-aversion or the dividend underestimation was the driver of this increase cannot be determined from this data.

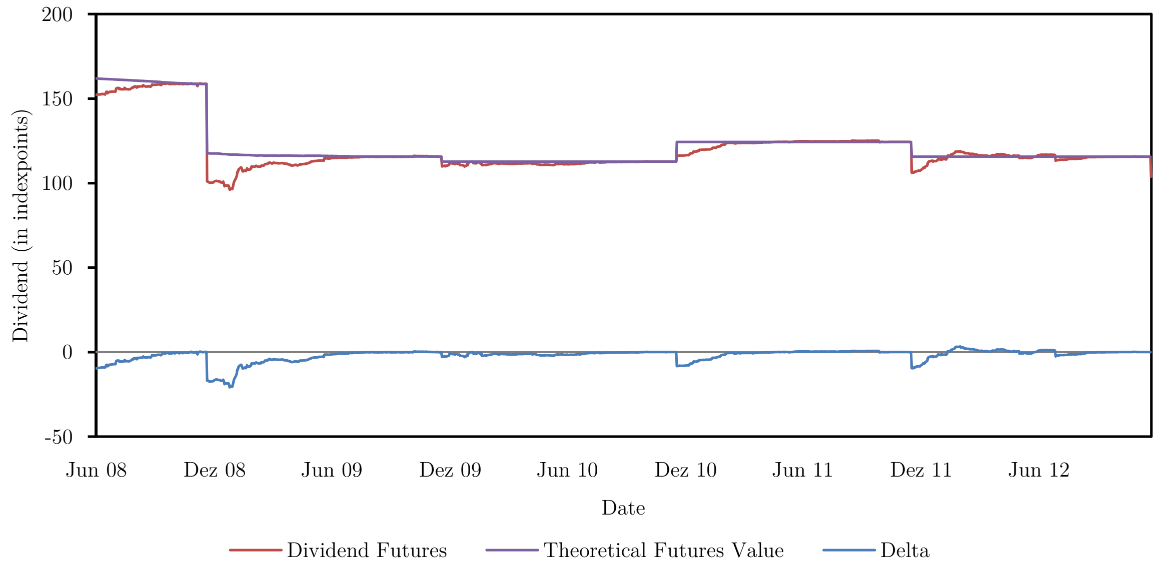
The lower value for G_t is a puzzle at first glance, but can be explained by lower dividend volatility on very short horizons. Thus, a more appropriate way to measure G_t is with long-

[21] http://aida.wss.yale.edu/~shiller/data.htm

term market valuations of future dividends. With the dividend futures of the EURO STOXX 50, this might be possible in the future, but more data are needed for a reliable measurement, otherwise risk aversion cannot be separated from dividend estimation errors. The dividend futures has as a payoff the realized dividend of a certain year. At the beginning of the year, the uncertainty is greater, as dividends are uncertain. The uncertainty decreases during the year, as an increasing number of dividends have already been paid, and the time to the payment of the unrealized dividends decreases. Thus, at the beginning of the year, these futures are also a good measurement for the market estimation of the dividends.

Figure 13 presents the price of the dividend futures and its theoretical value[22] from June 2008 to December 2012. The delta had its peak in January 2009, with 20.81 index points. This resembles the delta of 17.11 for the implicit dividend between March 2009 and December 2008 EURO STOXX 50 Futures in Figure 10. For the dividend future, the delta is a little higher, because it also contains the dividends from April to December 2009. Thus, this confirms that either the dividends were underestimated at the beginning of 2009, or risk aversion increased during that period, resulting in the drop in Figure 10 (and probably also in Figure 9).

Figure 13: EURO STOXX 50 Dividend Futures



This figure presents the price of the EURO STOXX 50 Dividend Futures compared to the theoretical price of that dividend future, using the compounded value of the actually realized dividends. The dividend futures is based on the yearly dividend amount. Delta is the difference between the dividend futures price and its theoretical price.

[22] The theoretical value of the dividend futures is merely the sum of the dividends in that year, compounded to the maturity of the future.

III.4 Replication of a Global Synthetic Investment

III.4.1 Portfolio Construction

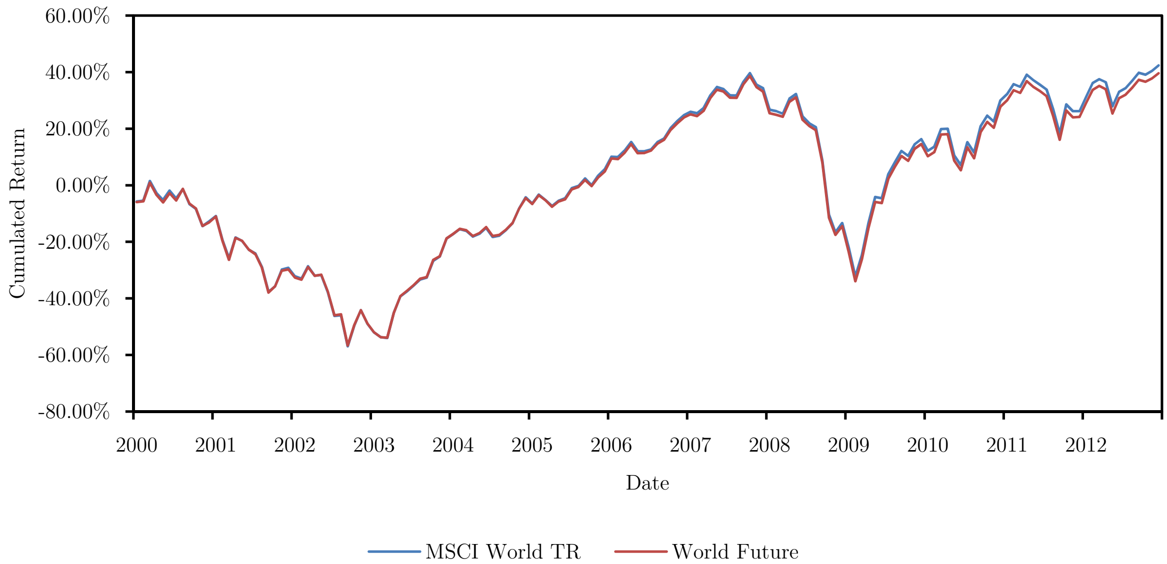
For each country in the MSCI World, the futures with the lowest tracking error is used as a replication instrument. The weight of the futures is basically the same as the weight of the corresponding country in the MSCI World. However, for some countries, no futures exist. In such cases, the weight of the other futures increases proportionally, so that the sum of the futures weight is always 100%. For each future, a currency forward, buying the currency of the futures and selling the domestic currency (USD), is done using the same weight as that of the future. The collateral is invested in risk-free USD.

The futures contracts and the forwards are rolled-on one day before the last trading date, since the liquidity of the futures decreases on the expiry date and the futures price is not the closing price of the index for all indices. The futures/forward portfolio is rebalanced monthly. This means that, every month, the weights of the futures are adjusted to the current MSCI World weight.

III.4.2 Monthly Returns

Figure 14 shows the cumulated return of the MSCI World gross return compared to the futures portfolio in USD for the period from January 2000 to December 2012.

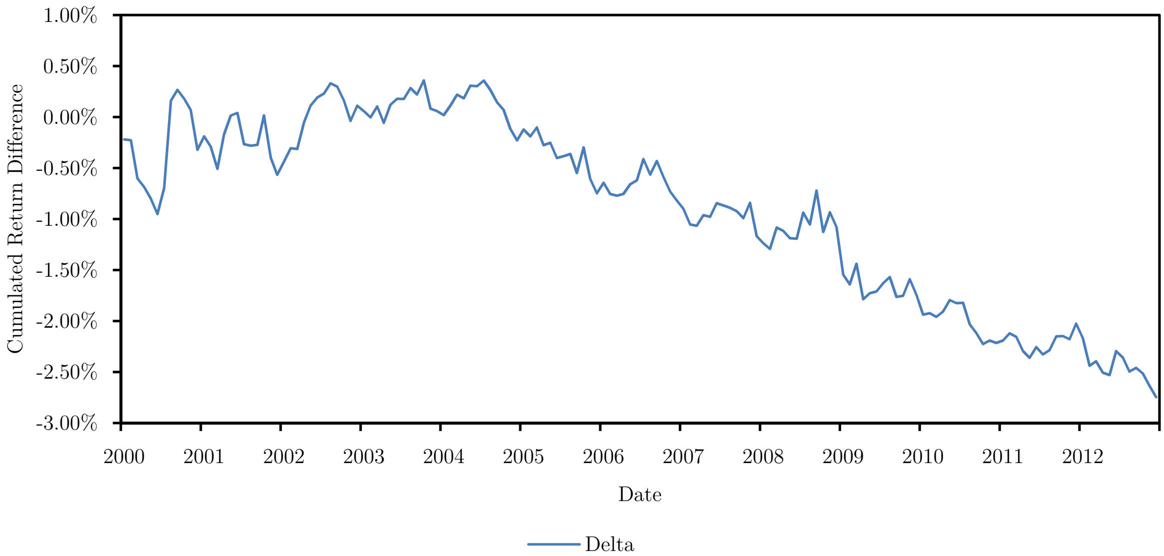
Figure 14: Cumulated Futures Return versus the MSCI World



This figure presents the cumulated return of the MSCI World total return index compared to the return of a futures portfolio replicating that index on a monthly basis. Each futures is weighted according to the weight of its country in the MSCI World. In the event that no futures was available for a month, all other futures were scaled up to the sum of 1. The period is from January 2000 to December 2012.

The development of both returns is almost identical. Figure 15 shows the difference between the futures portfolio and the MSCI World index, so as to reveal the development in more detail.

Figure 15: Cumulated Return Difference between Futures and MSCI World



This figure presents the cumulated return difference between the world futures portfolio and the MSCI World total return index on a monthly basis. Each futures is weighted according to the weight of its country in the MSCI World. In the event that no futures was available for a month, all other futures were scaled up to the sum of 1. The period is from January 2000 to December 2012.

The futures portfolio generates an annual return of 3.05%, and the MSCI a gross return 3.26%; thus, the futures return is -0.21% below the MSCI gross return index. However, the MSCI net return index is 2.77%, which means the futures return is 0.28% above the MSCI net return index.

III.4.3 Tracking Error

The tracking error between the index gross return and the MSCI World gross return is 0.58% p.a. This is the error that is caused by different constituents. The tracking error between the futures and the index gross return is 0.43%, due to futures mispricing and different closing times. Finally, the tracking error between the futures and the MSCI World gross return index is also 0.58%.

III.4.4 Transaction Costs

The transaction costs of the futures portfolio and the MSCI portfolio with the MSCI weightings of December 31 2012 are summarized in Table 11.

Table 11: Transaction Costs Futures versus MSCI

	Synthetic Replication	MSCI World
Brokerage Fee	0.5	3.0
Transaction Tax	-	2.7
1/2 BA Spread	1.6	3.7
1/2 BA FX	1.1	1.1
Total Initial Costs	3.3	10.5
Rollover Costs	11.8	0
Dividend Reinvestment	0	0.5
Index Change	0	0.5
Total Annual Maintenance Costs	11.8	1.0

This table presents the constituents of transaction costs for a cash MSCI World investment compared to the costs for a synthetic investment with futures. The weightings are those of December 31 2012.

These results show that the initial costs of establishing a futures portfolio that replicates the MSCI World index are less than a third of the costs to buy the cash stocks. However, the annual maintenance costs are much higher, since the futures must be rolled over quarterly or monthly.

III.4.5 Constituents of Futures Portfolios

Table 12 shows the figures of a futures portfolio with different numbers of futures, in order to replicate the MSCI World. One futures means that only the futures whose country has the highest weight in the MSCI World is used to replicate the MSCI World portfolio, while two futures means that the futures of the two biggest countries are taken.

A complete futures portfolio is not the best solution, due to the high rollover costs. Reducing the number of futures from 21 to 15 reduces the annual costs from 11.5 to 7.8 bps, while the tracking error increases slightly from 58 bps 67 bps and the return decreases by 8 bps. Using 10 futures reduces the return by 13 bps p.a., while the tracking error increases by 43 bps, from 58 to 101 bps. The initial costs decrease slightly, from 3.2 bps to 2.8 bps, while the maintenance costs decrease by almost 50% from 11.5 bps to 6.7 bps.

Table 12: Different Futures Portfolios

	Return	Tracking Error	Initial Costs	Annual Costs
1 Future	-0.67%	4.13%	1.3	2.2
2 Futures	-0.55%	3.14%	2.3	3.8
3 Futures	-1.31%	2.77%	2.4	4.0
4 Futures	-1.01%	2.54%	2.6	4.1
5 Futures	-0.95%	1.92%	2.6	4.3
6 Futures	-0.66%	1.70%	2.7	4.5
7 Futures	-0.55%	1.26%	2.7	4.5
8 Futures	-0.42%	1.12%	2.7	4.4
9 Futures	-0.37%	1.06%	2.7	4.6
10 Futures	-0.34%	1.01%	2.8	6.7
11 Futures	-0.28%	0.89%	2.8	6.8
12 Futures	-0.29%	0.76%	2.8	7.0
13 Futures	-0.32%	0.70%	2.8	7.1
14 Futures	-0.30%	0.69%	2.8	7.7
15 Futures	-0.29%	0.67%	2.9	7.8
16 Futures	-0.26%	0.66%	3.0	10.0
17 Futures	-0.23%	0.63%	3.1	10.2
18 Futures	-0.24%	0.59%	3.1	11.2
19 Futures	-0.24%	0.59%	3.2	11.4
20 Futures	-0.23%	0.59%	3.2	11.4
All Futures	-0.21%	0.58%	3.2	11.5

This table presents a return and tracking error analysis for a futures portfolio compared to the MSCI World total return index. The first column indicates how many futures are combined in order to replicate the MSCI World. The futures are selected according to their weight in the MSCI world on December 31 2012, so that for one future, that with the largest country weight in the MSCI world is selected. For two futures, the largest and the second largest are selected. The second column presents the return difference between the futures portfolio and the MSCI World index. The third column presents the tracking between the futures portfolio and the MSCI World index. The fourth column indicates the transaction costs to initiate the futures portfolio, while the last column indicates the transaction costs to maintain such a futures portfolio. The period is from January 2000 to December 2012.

Table 13 shows this analysis, replacing all futures in the Eurozone with the EURO STOXX 50 futures. These results show that the EURO STOXX 50 Futures have the advantage of using one futures to capture a large market. If the same number of futures is used, the tracking error of the basket including the EURO STOXX 50 Futures is always lower, compared to the basket excluding the EURO STOXX 50 Futures. The transaction costs are higher, due to the relatively high bid-ask spread of the EURO STOXX 50 Futures, caused by a relatively high tick size. The EURO STOXX 50 Futures achieve good results if only a few futures are demanded, so example 5-7 futures. In this area, transaction costs are four times lower than they are in cash stocks, and annual costs are only 5.4 bps. The tracking error of only five futures is 1.09%, which means taking five futures will have a 4 times lower transaction cost, only 5.3 bps annual rollover costs, while the cash portfolio invests in 1700 stocks.

The optimal solution for investors who want to use only a few futures is in the area of 5-7 futures, including the EURO STOXX 50 Futures. Someone who is not concerned with the number of futures would do better to ignore the EURO STOXX 50 Futures and should rather obtain around 15 futures to replicate the MSCI World portfolio with relatively low transaction costs and tracking errors.

Table 13: Different Futures Portfolios, Including the EURO STOXX 50 Futures

	Return	Tracking Error	Initial Costs	Annual Costs
1 Future	-0.67%	4.13%	1.3	2.2
2 Futures	-0.65%	2.73%	1.8	4.3
3 Futures	-0.54%	2.19%	2.5	5.3
4 Futures	-1.17%	1.34%	2.5	5.3
5 Futures	-0.92%	1.09%	2.7	5.3
6 Futures	-0.66%	0.93%	2.8	5.4
7 Futures	-0.53%	0.85%	2.8	5.4
8 Futures	-0.48%	0.82%	2.9	7.4
9 Futures	-0.45%	0.78%	2.9	7.5
10 Futures	-0.43%	0.76%	2.9	8.0
11 Futures	-0.40%	0.75%	3.1	10.2
All Futures	-0.37%	0.73%	3.1	11.2

This table presents a return and tracking error analysis for a futures portfolio compared to the MSCI World total return index. The MSCI Eurozone is replicated using the EURO STOXX 50 Futures instead of a futures for each country. The first column indicates how many futures are combined to replicate the MSCI World. The futures are selected according to their weight in the MSCI world on December 31 2012, so for 1 future, that with the largest country weight in the MSCI world is selected. For two futures, the largest and the second largest are selected. The second column presents the return difference between the futures portfolio and the MSCI World index. The third column presents the tracking between the futures portfolio and the MSCI World index. The fourth column indicates the transaction costs to initiate the futures portfolio, while the last column indicates the transaction costs to maintain such a futures portfolio. The period is from January 2000 to December 2012.

III.4.6 Investment Advice for Both Markets

The optimal futures portfolio depends on the trading activity, the relevance of the tracking error, the transaction costs, and the withholding taxes of the specific investor.

Let us assume we have three kinds of investors: firstly, a US investor, representing an investor of a large country, with a domestic currency of USD and who can completely reclaim the withholding tax in the US, but who can reclaim only 50% in the rest of the world. Secondly, a Swiss Investor, representing a small country, with a domestic currency of CHF and who can completely reclaim the withholding tax in Switzerland, but who can reclaim only 50% in the rest of the world. Thirdly, a global investor, which could be an international fund such as the iShares MSCI World ETF, and which cannot usually reclaim any withholding taxes and is based in USD.

Table 14 shows the analysis for these three investors, using the 15 largest futures. All investors have annual transaction costs of 7.8 bps and a tracking error of 67 bps against the MSCI World portfolio if they take the 15 largest futures. Furthermore, the historical loss of those futures against their total return index was 29.1 bps, which is 8 bps higher than in chapter III.4.2, because here we use only 15 futures.

The initial transaction costs are 2.4 bps for the US investor and for the global investor, as both have USD as their domestic currency and do not have currency costs on 50% of the portfolio that is invested in USD. The initial costs for the Swiss investor are 3.5 bps, which is higher because s/he has higher currency transaction costs. S/he has to change 96% of the portfolio into foreign currency, and the currency bid-ask spreads against CHF are slightly higher than they are for USD.

Table 14: Transaction Costs for Different Investors

	US Investor	Swiss Investor	Global Investor	MSCI World
Annual Transaction Costs	7.8	7.8	7.8	1.0
Historical Futures Loss	29.1	29.1	29.1	
Implicit WHT Benefit	14.1	31.8	66.1	
Total Annual Costs	22.8	5.1	-29.2	1.0
Initial Costs	2.4	3.5	2.4	10.0
Tracking Error	0.67%	0.67%	0.67%	0.00%

This table presents different kinds of transaction costs in basis points. The transaction costs for different investors in futures are listed in columns 2-4 and for a cash MSCI World portfolio in the last column. The historical futures loss happens in the period from January 2000 to December 2012

The biggest difference is generated by the implicit withholding tax benefit. The withholding tax benefit is actually the withholding tax costs that are incurred by the cash investor in the MSCI world. The futures investor does not pay these costs, as s/he can invest the collateral from the futures domestically. Since the futures investor does not pay these costs, it is an implicit benefit for him or her. For the US investor, the benefit is the lowest, with 14.1 bps, because s/he cannot benefit in the US market because s/he could reclaim all withholding taxes completely in the cash market. For the Swiss investor, the benefit is 31.8 bps, since s/he could reclaim only the Swiss taxes in full. For the global investor, the benefit is 66.1 bps and is thus the highest of all, because s/he cannot reclaim any withholding taxes and pays the full rate in every country.

For all investors, it is only worth using the futures portfolio to replicate the MSCI world if they can accept a tracking error of 67 bps p.a. A passive investor with lower tracking error permission cannot buy a complete futures portfolio. However, s/he could use a fraction of his or her portfolio to invest in futures in order to manage short-term transactions, which could indicate a fund manager with in- and outflows of his or her fund.

How long does it take to achieve a cost equivalence of futures and cash investments?

US investor: For the US investor, the cost equivalence of futures and cash is after 4 months. Thus, the round-trip transaction costs are equal after 8 months. This means that the US investor should conduct all transactions that have a horizon shorter than 8 months with futures. Alternatively, s/he could use futures for global investments and invest in his or her home cash market.

Swiss investor: For the Swiss investor, the cost equivalence of futures and cash market is after 1.5 years. Thus, the round-trip transaction costs are equal after 3.0 years. This means that the Swiss investor should conduct all transactions that have a horizon shorter than 3.0 years with futures. Only long-term investments of more than 3.0 years should be done via cash investments.

Global investor: For the global investor who loses all withholding taxes with the cash investment, it is always better to invest in futures. With the futures portfolio, s/he gains 29.2 bps against the cash portfolio every year.

III.5 Concluding Remarks

This paper has analyzed the equity futures in comparison to the cash equity market from different perspectives. Overall, the futures market is highly efficient from every perspective. There have been some arbitrage opportunities from time to time, but only for small markets. Futures can also be used to observe the market estimation of the future dividends. Even this has proved to be very efficient, with a major exception in 2008. Futures implied dividends show that the market has underestimated the subsequent realized dividends, or increased risk-aversion has caused a lower market value of future dividends. However, the analysis also proves that tax effects did not cause this deviation, as the tax-free futures FTSE 100 exhibits the same underestimation.

Given the efficiency of the futures pricing, futures offer two main advantages. Firstly, transaction costs are, on average, one third less for futures investments than they are in the cash index. However, this benefit decreases with time, as the ongoing costs of futures are higher because they have to be rolled-over periodically. Secondly, futures have an implicit withholding tax benefit, as dividends are received implicitly with the futures price and are not taxable, while withholding taxes have to be paid in the cash market and many investors cannot completely reclaim them in foreign countries. Thus, it depends on the tax status of the investor as to whether it is better to invest solely in futures, or whether a futures investment should only be for a limited time, as ongoing costs eat up the initial transaction cost benefit.

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